

Greedy consumer in the Rosenzweig-MacArthur model

Przemysław Gawroński¹, Alfio Borzi², Krzysztof Kułakowski¹



¹AGH University of Krakow, Poland

²Universität Würzburg, Germany



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abstract

The system of one consumer and two resources is investigated numerically within the Rosenzweig-MacArthur model with a Holling type 2 functional response. Here the model equations are supplemented by the normalization of the rates of consumption to unity. This reflects the limited amount of time which can be devoted to a given type of resource. Further, the consumer is allowed to modify the rates as to enhance the consumer density. This process can lead to an absorbing state where one of resource is extinct; then, the diversity is deteriorated. The results of numerical calculations allow to identify transient chaos of the time, before the stationary state is reached. In this state the system shows a sequence of Hopf bifurcations and linear variations in time. This stationary state is shown to preserve an information of the initial state of the rates of consumption.

keywords: consumption, nonlinearity, chaos, ecology, differential equations

outline

1. Evolutionary game theory : lack of information, trembling hand, learning
2. Deterministic models: Lotka-Volterra, Kuramoto, Rosenzweig-MacArthur models
3. Fixed points and periodic orbits. Synchronization.
4. New ingredients of the RM model
5. Results: dynamics in symmetric, asymmetric and absorbing states
6. Results: transient chaos and chirality
7. Results: Fourier spectrum and memory effect
8. Conclusions: greediness, diversity, instability

Population dynamics is the portion of ecology that deals with the variation in time and space of population size and density for one or more species (Begon et al., 1990, [5].)

Link to FENS:

evolutionary game theory (imperfect information, learning).

Method:

ordinary non-linear differential equations, numerical solutions.

Lotka-Volterra model

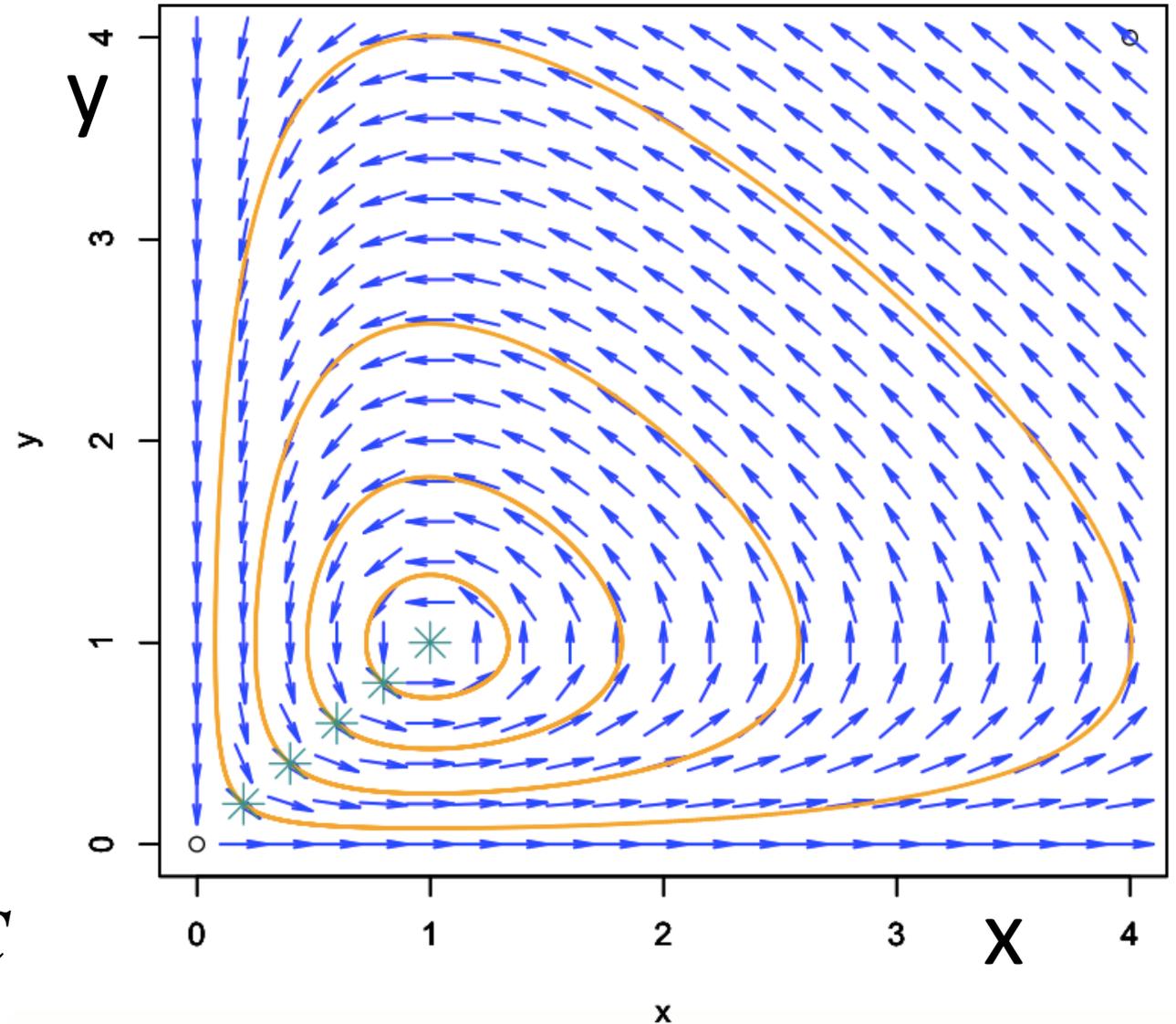
$$\frac{dx}{dt} = r(1-y)x$$

$$\frac{dy}{dt} = m(x-1)y$$

Drawbacks:

$$x(t, y = 0) \rightarrow \infty$$

$$m(x - \ln(x)) + r(y - \ln(y)) = C$$



Lotka-Volterra model, „atto-fox” problem

$$\frac{dx}{dt} = 3(1-y)x$$

$$\frac{dy}{dt} = (x-1)y$$

Starting from
 $(x,y)=(0.01,0.01)$
we get

$$x_{\min} \approx 10^{-8}$$

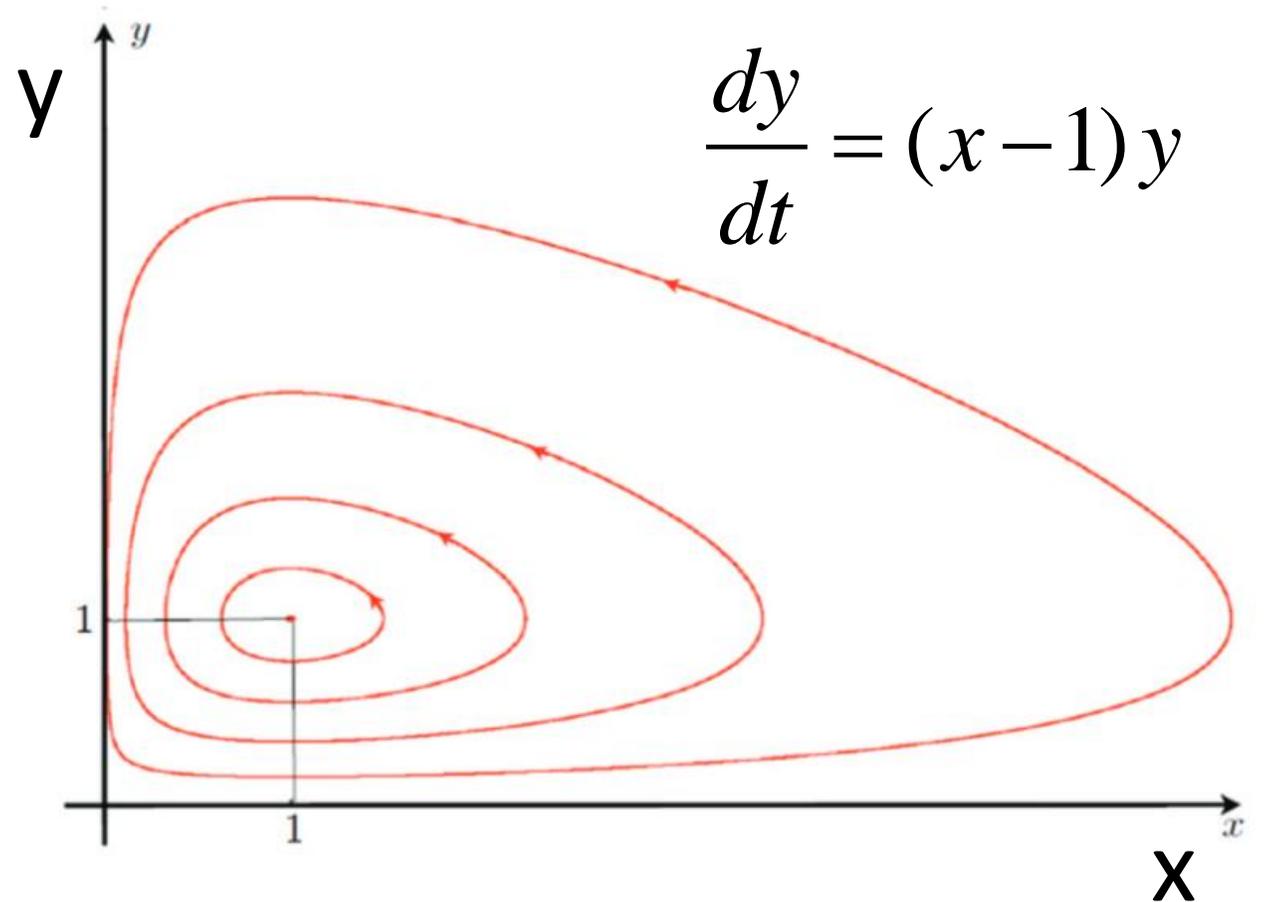


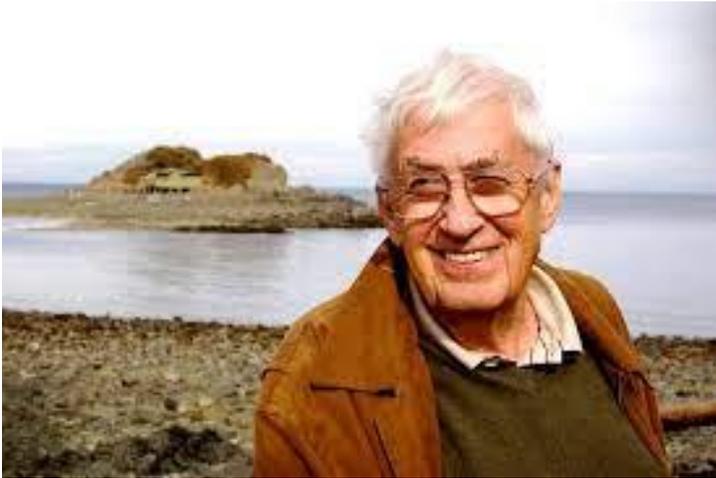
Figure 1.4. Simulations of system [1.23]

[Claude Lobry, 2018]

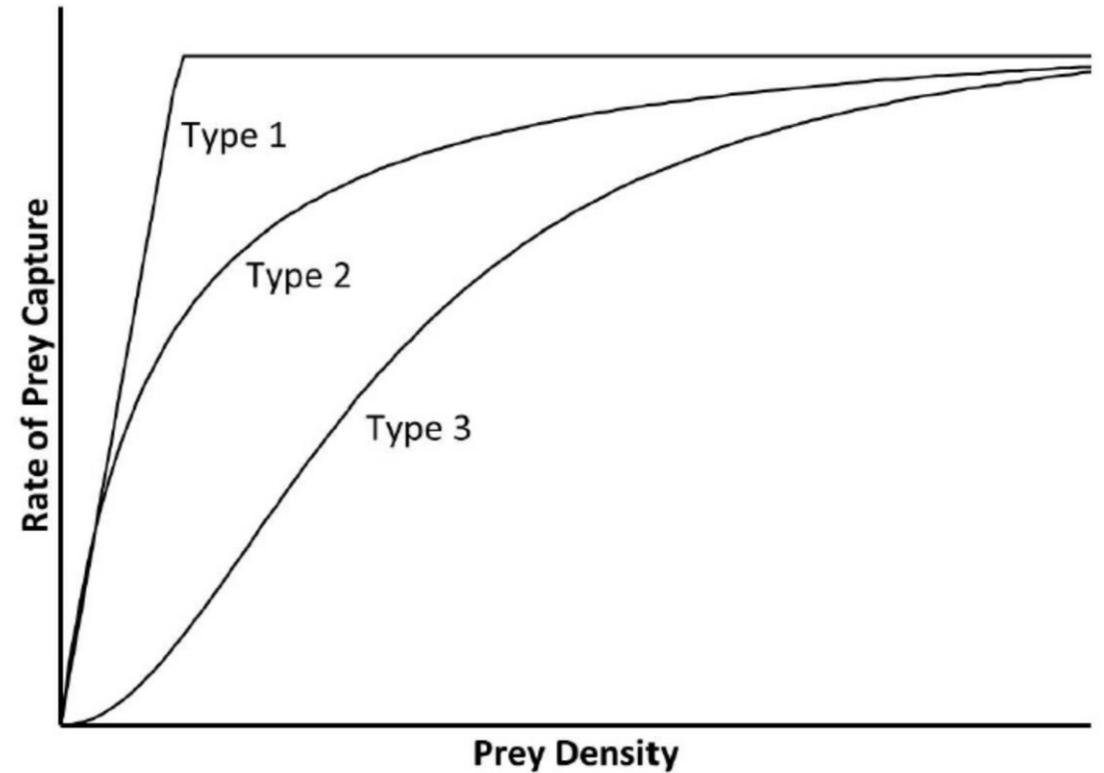
RM model of type 2: one resource R , one consumer C (1C1R)

$$\frac{dR}{dt} = R(1 - R) - \frac{pRC}{1 + bR}$$

$$\frac{dC}{dt} = \frac{pRC}{1 + bR} - mC$$



Crawford Stanley "Buzz" Holling, 1930-2019



Types I, II, and III functional responses [1]

Fixed point

$$\frac{dR}{dt} = 0$$

$$R^* = \frac{m}{p - mb}$$

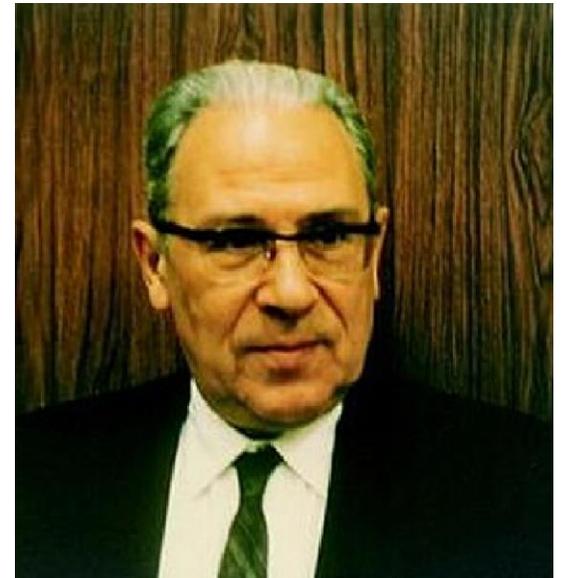
$$\frac{dC}{dt} = 0$$

$$C^* = \frac{p - m(b + 1)}{(p - mb)^2}$$

Stability conditions: eigenvalues of the Jacobian negative

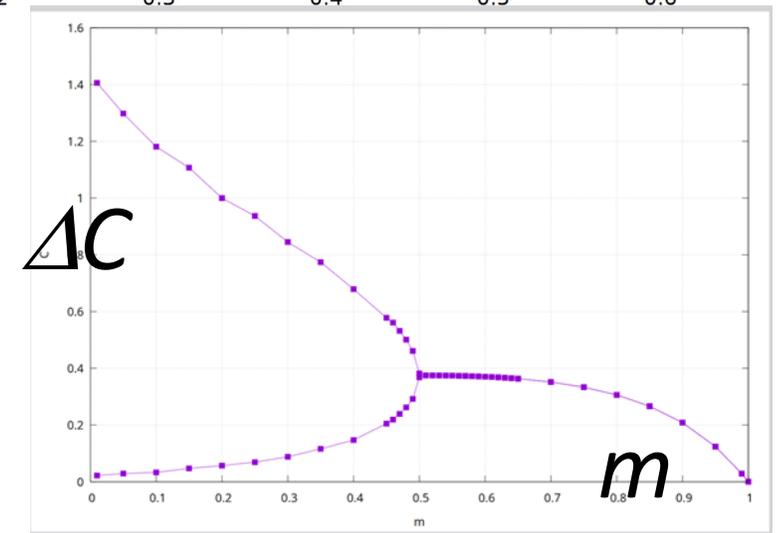
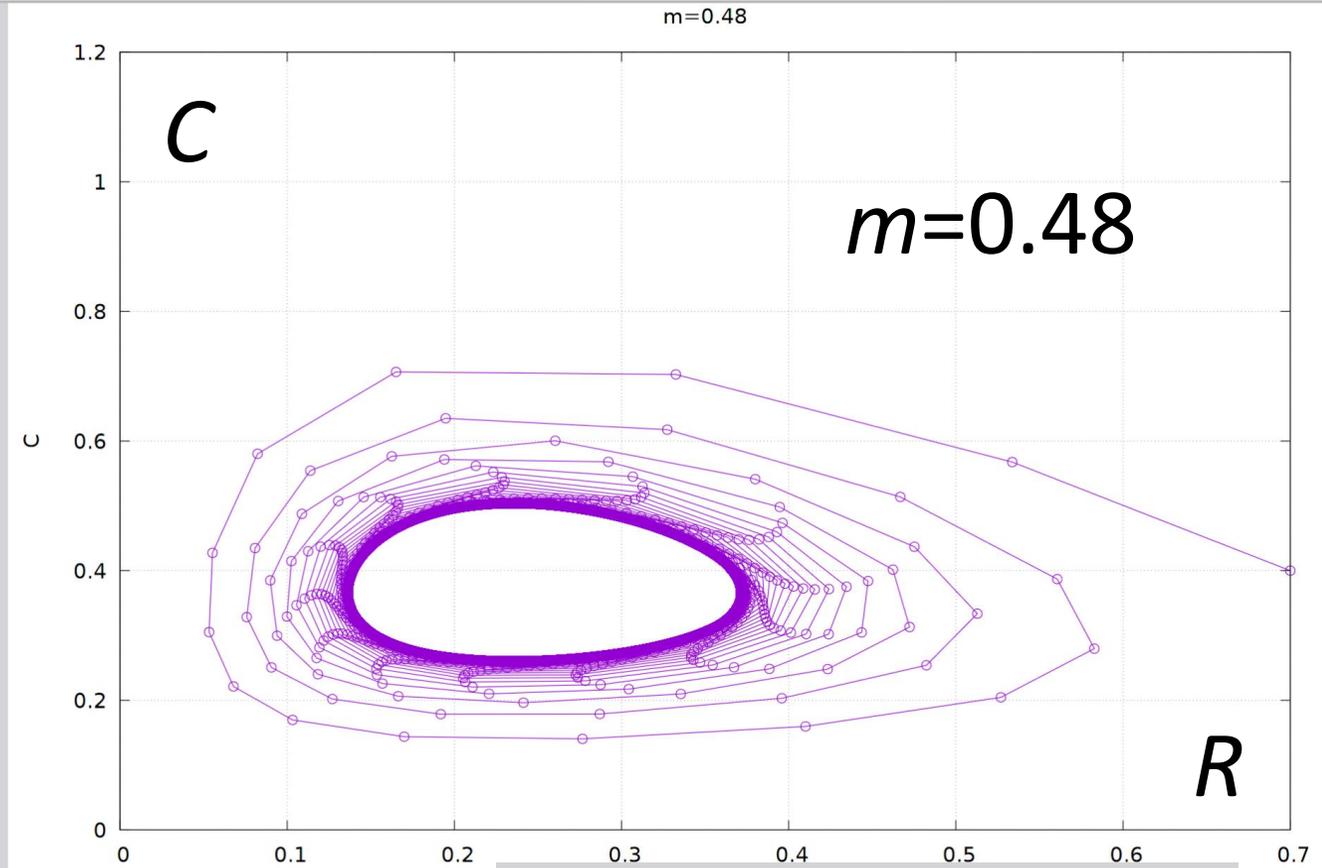
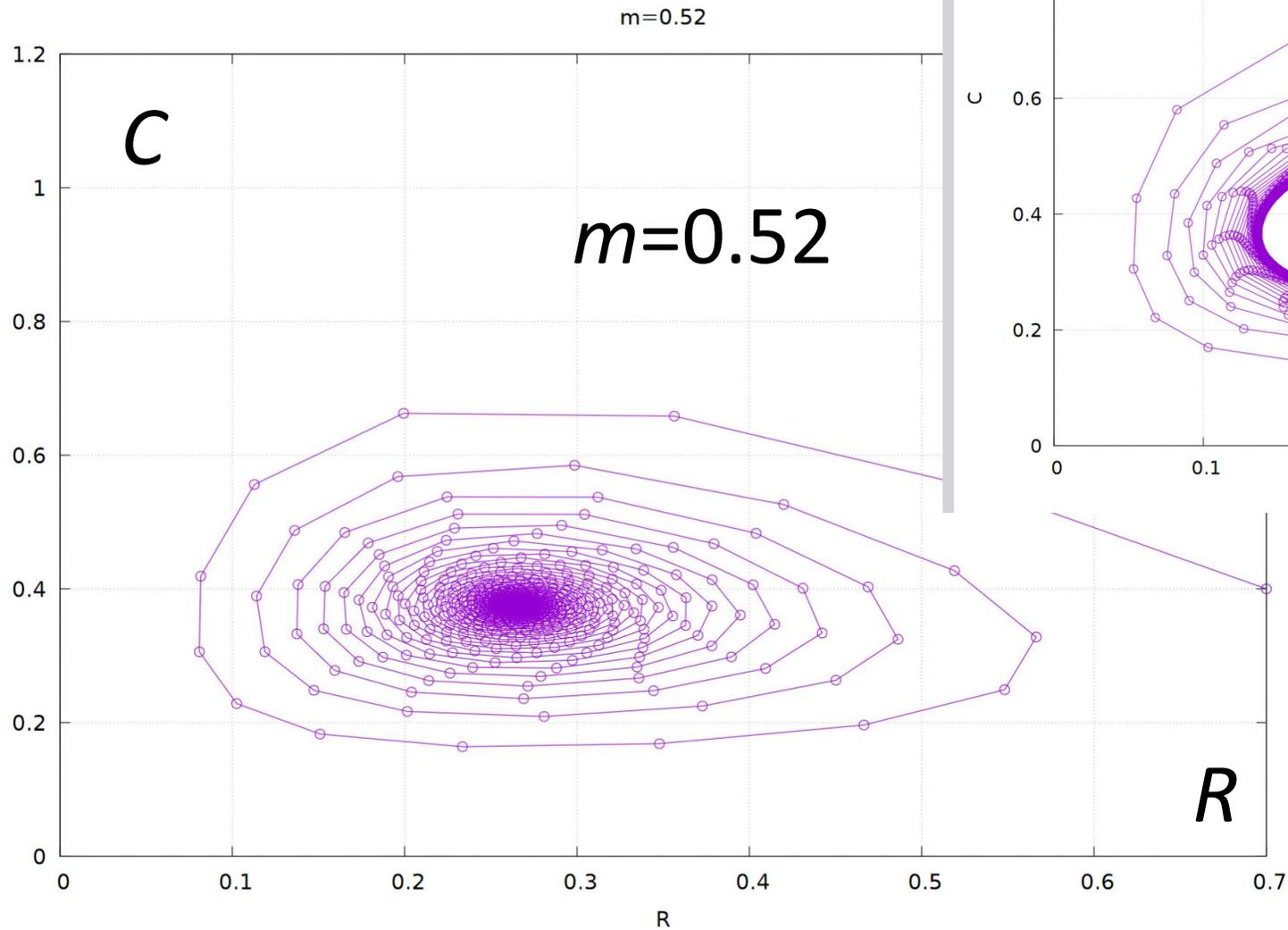
$$p - mb > 0 \quad m > \frac{p(b - 1)}{b(b + 1)}$$

For $p=3.0$, $b=2.0$ the Hopf bifurcation at $m=0.5$

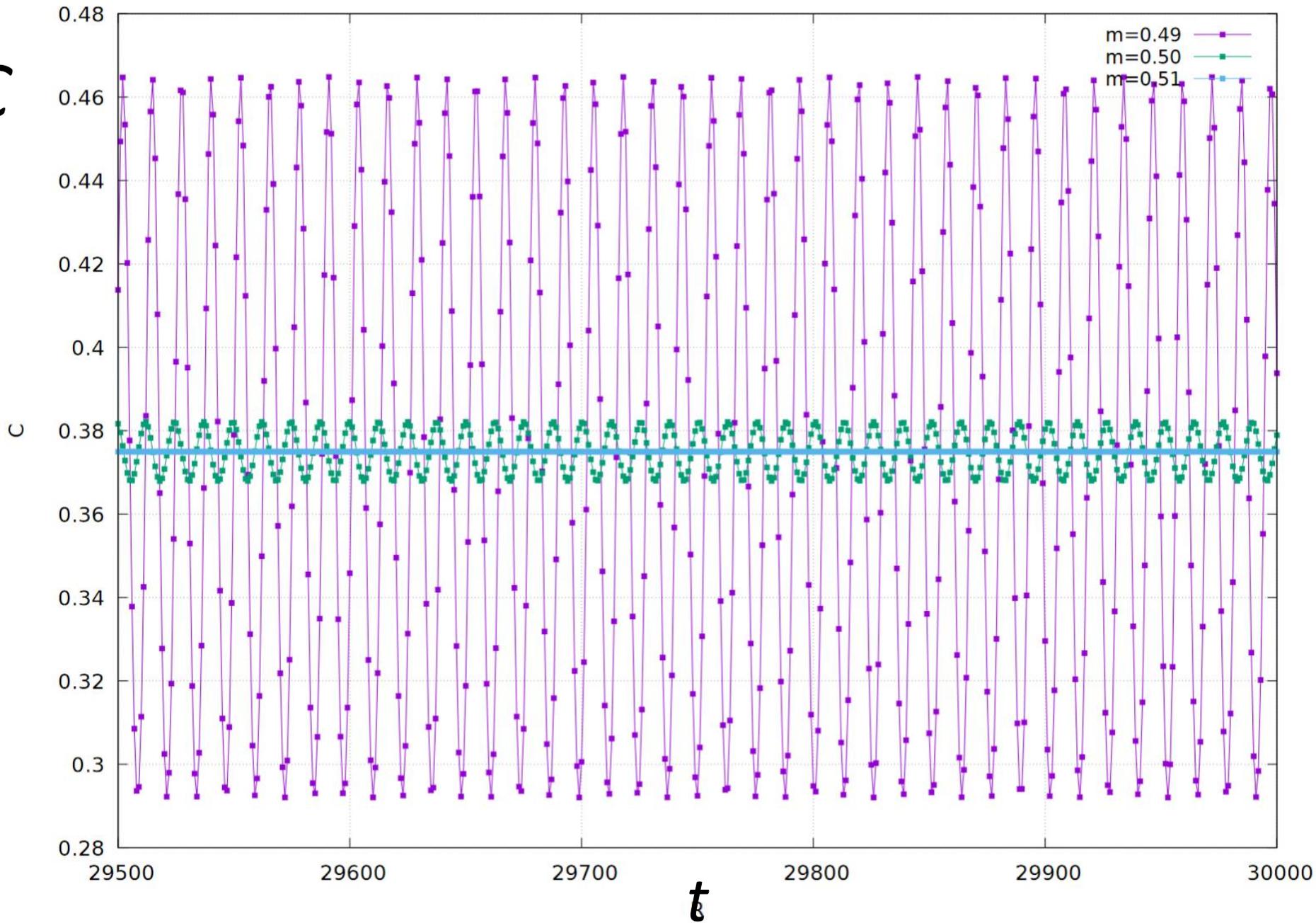


Eberhard Ferdinand Friederich Hopf, 1902-1993

RM 1C1R, Hopf bifurcation



C



RM model: two resources R_i , one consumer C (1C2R) [2]

$$\frac{dR_1}{dt} = R_1(1 - \alpha_{11}R_1 - \alpha_{12}R_2) - \frac{pR_1\beta_1C}{1 + b(\beta_1R_1 + \beta_2R_2)}$$

$$\frac{dR_2}{dt} = R_2(1 - \alpha_{21}R_1 - \alpha_{22}R_2) - \frac{pR_2\beta_2C}{1 + b(\beta_1R_1 + \beta_2R_2)}$$

$$\frac{dC}{dt} = pC \frac{\beta_1R_1 + \beta_2R_2}{1 + b(\beta_1R_1 + \beta_2R_2)} - mC$$

The planes $R_i = 0$, $C = 0$ are **absorbing**.

Our choice: $p=3.0$, $b=2.0$, $m=0.2$, $\alpha_{12} = \alpha_{21} = 0.065$, $\alpha_{11} = \alpha_{22}$

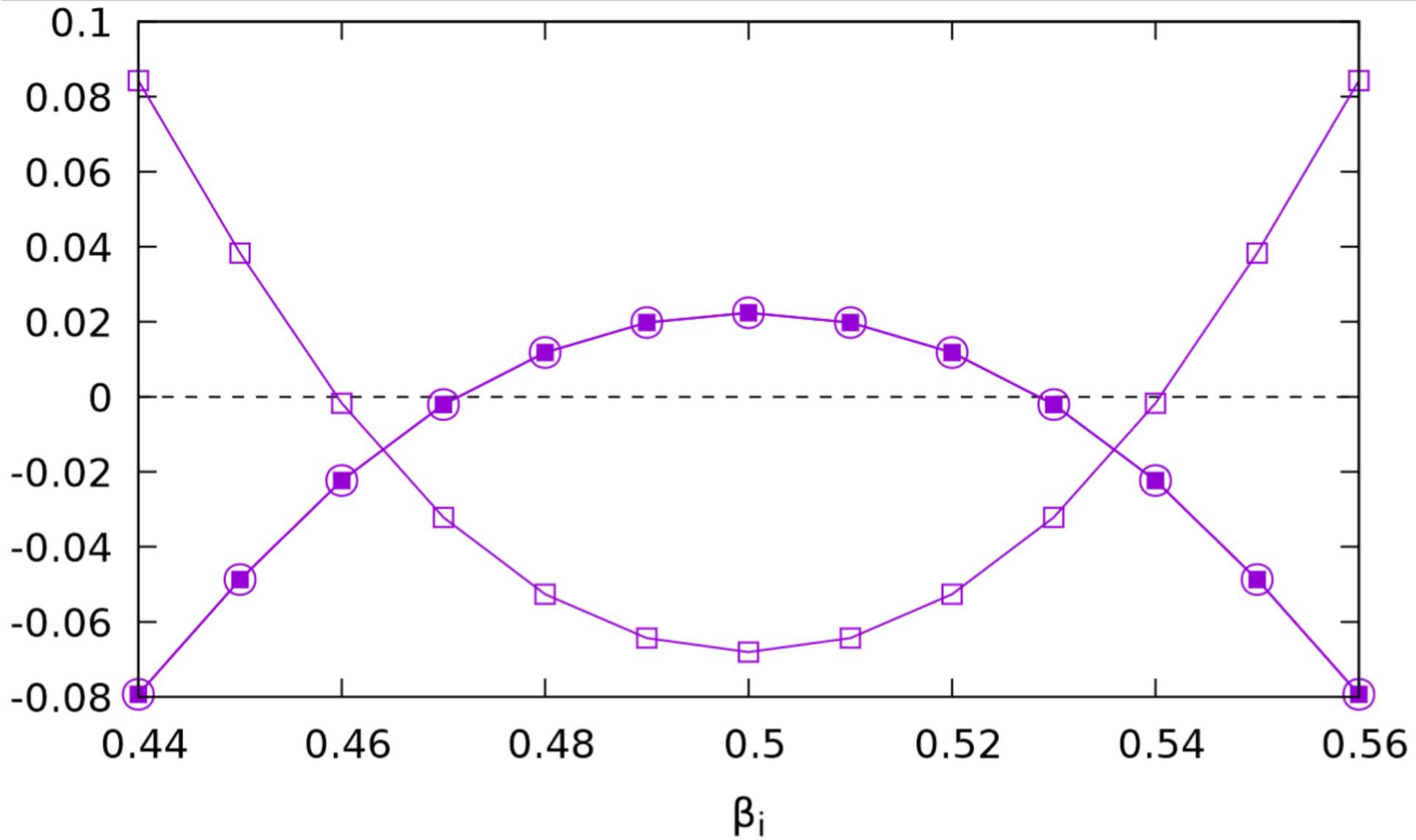
Assumption I [3]:

The rate of consumption is limited; the consumer should make a choice between R_1 and R_2 :

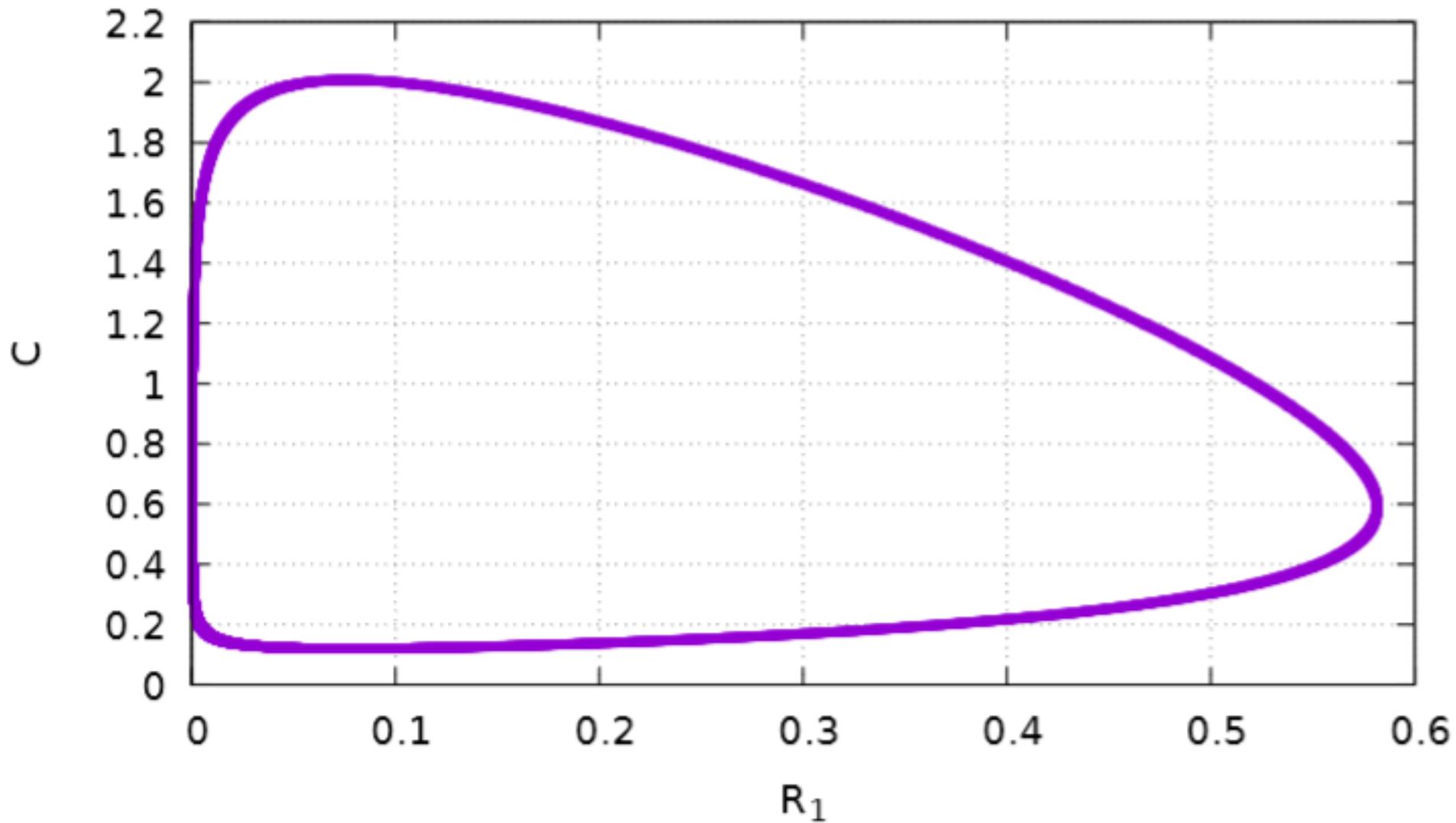
$$\beta_1 + \beta_2 = 1$$

Then, the system is symmetric : $\beta_i \Leftrightarrow 1 - \beta_i$

and the point $\beta_i = 0.5$ is specific.



Three eigenvalues of the Jacobian for the RM model 1C2R.



The C-R periodic orbit for the RM 1C2R model, $\beta_i=0.5$. Here $R_1(t)=R_2(t)$.

assumption II [3]:

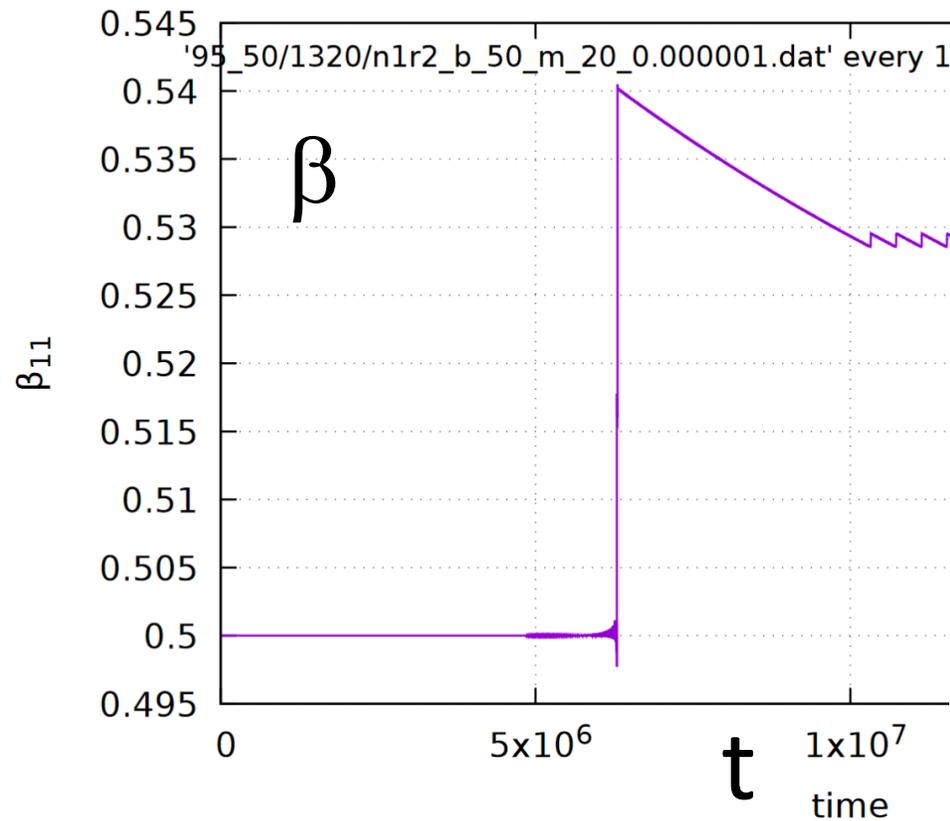
The consumer intends to modify the coefficients β_i as to increase C :

$$\frac{d\beta_i}{dt} = v \frac{dC}{d\beta_i}$$

where v is a measure of the greediness.

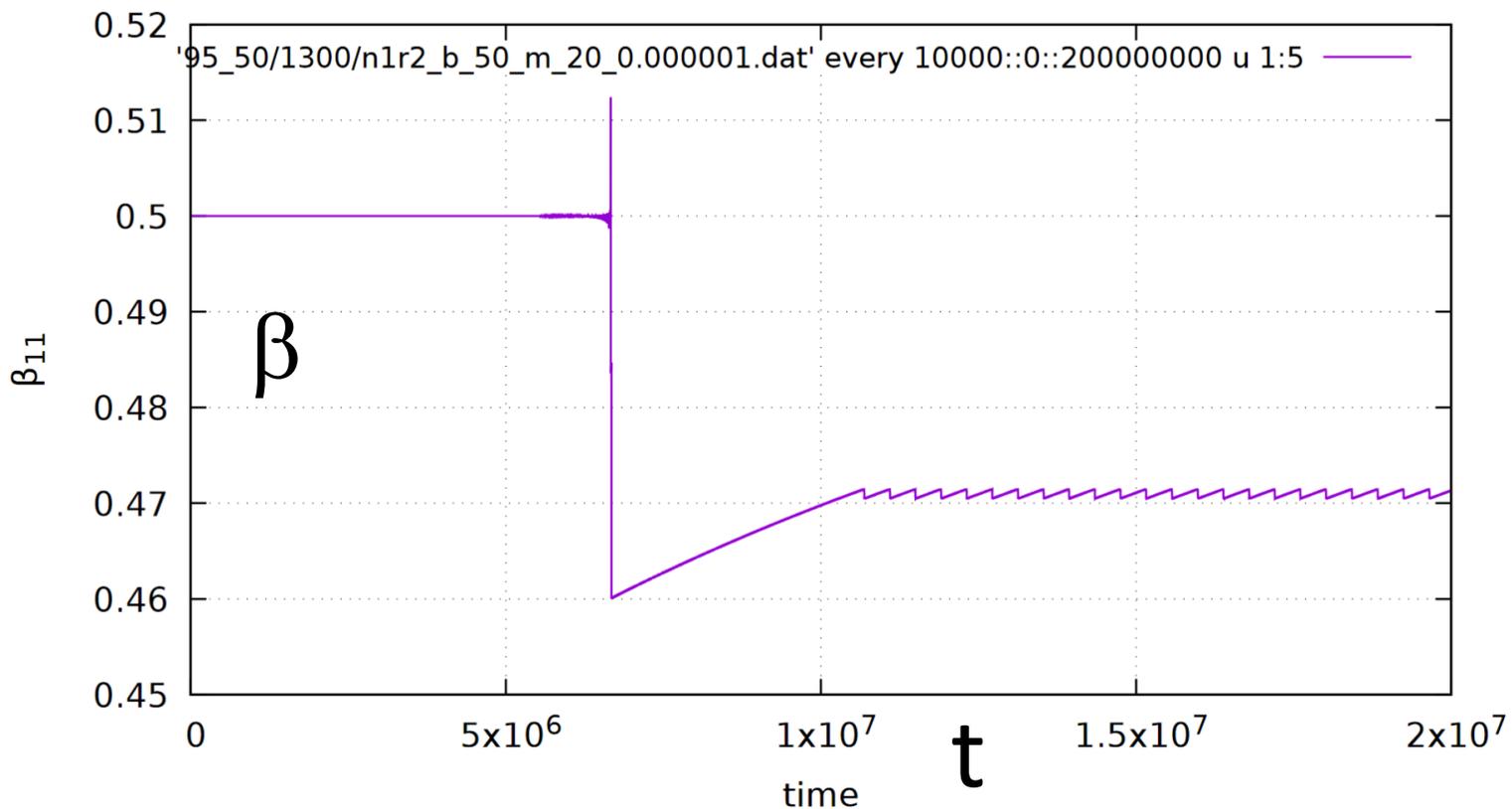
For $R_1 = R_2$, the set $\beta_i = 0.5$ is invariant...

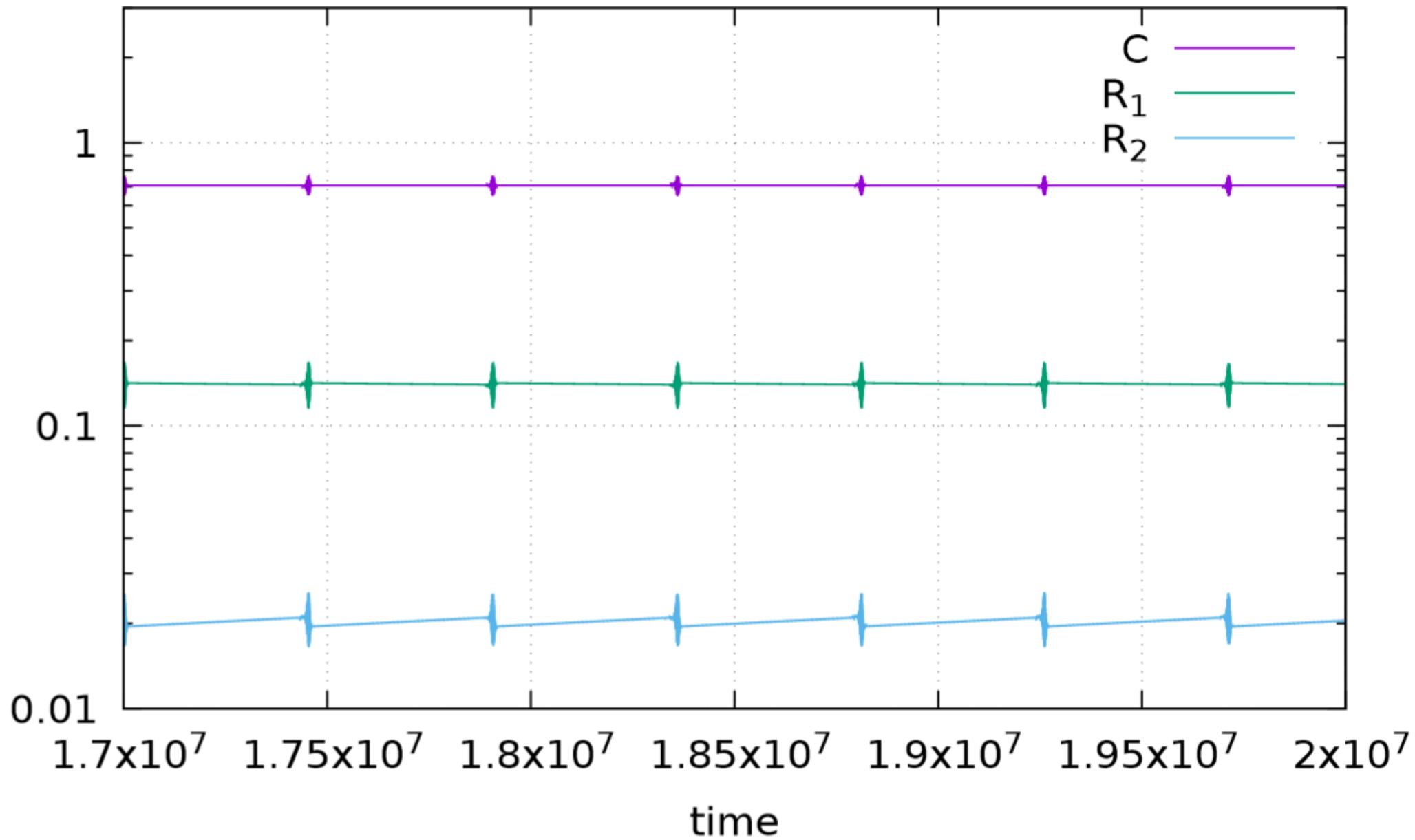
$m = 0.20, \alpha = (0.95, 0.065, 0.065, 0.95), \varepsilon = 10^{-6}, \beta_{11} = 0.5, \text{wsp} = 0.001320$



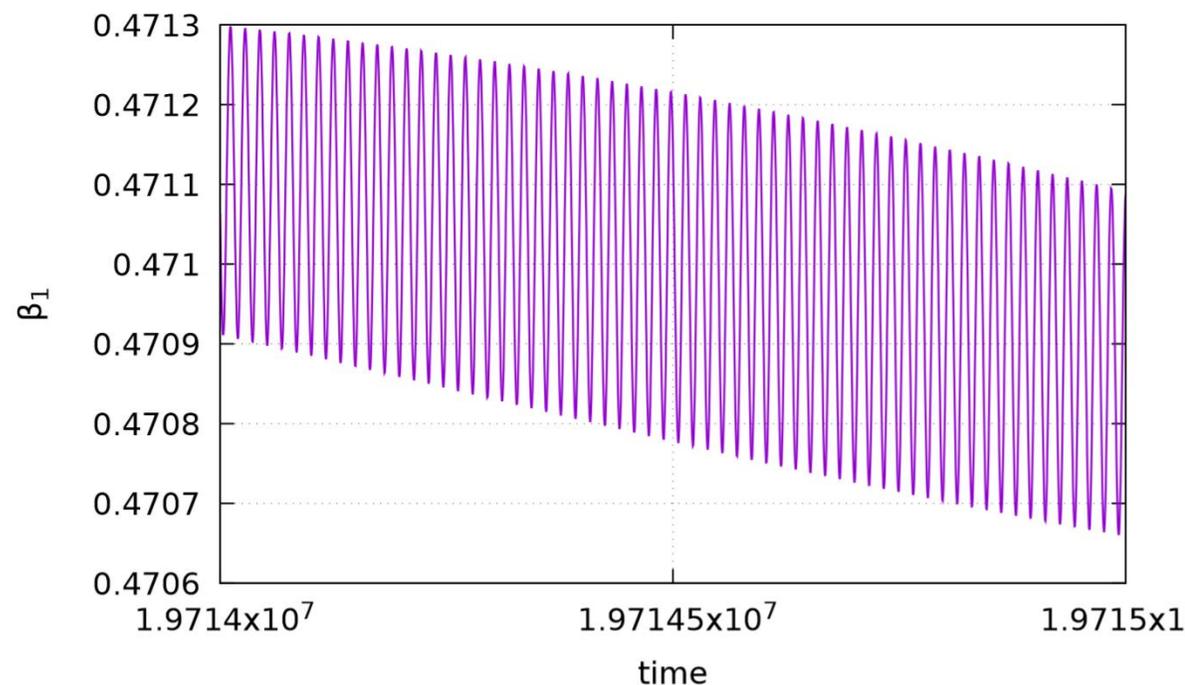
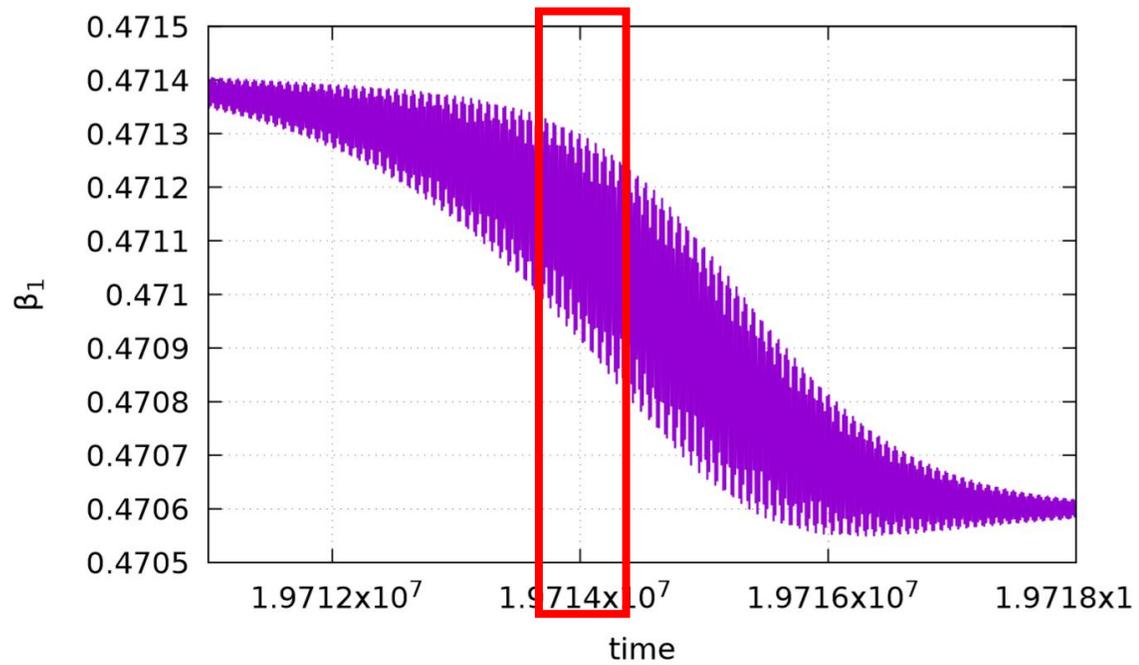
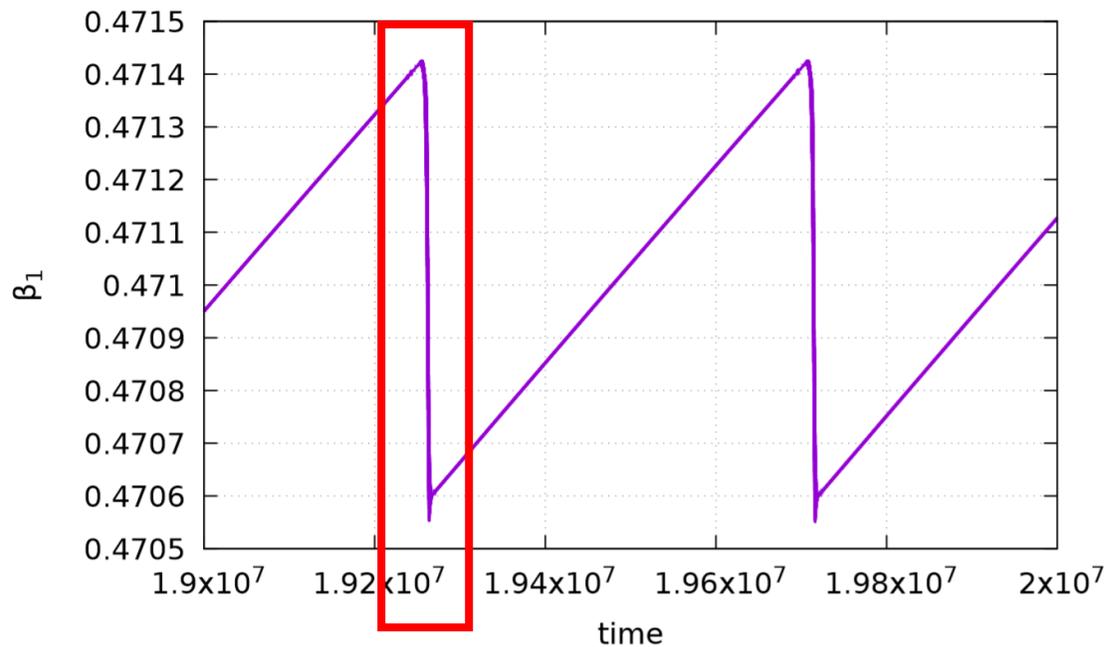
...but may be unstable

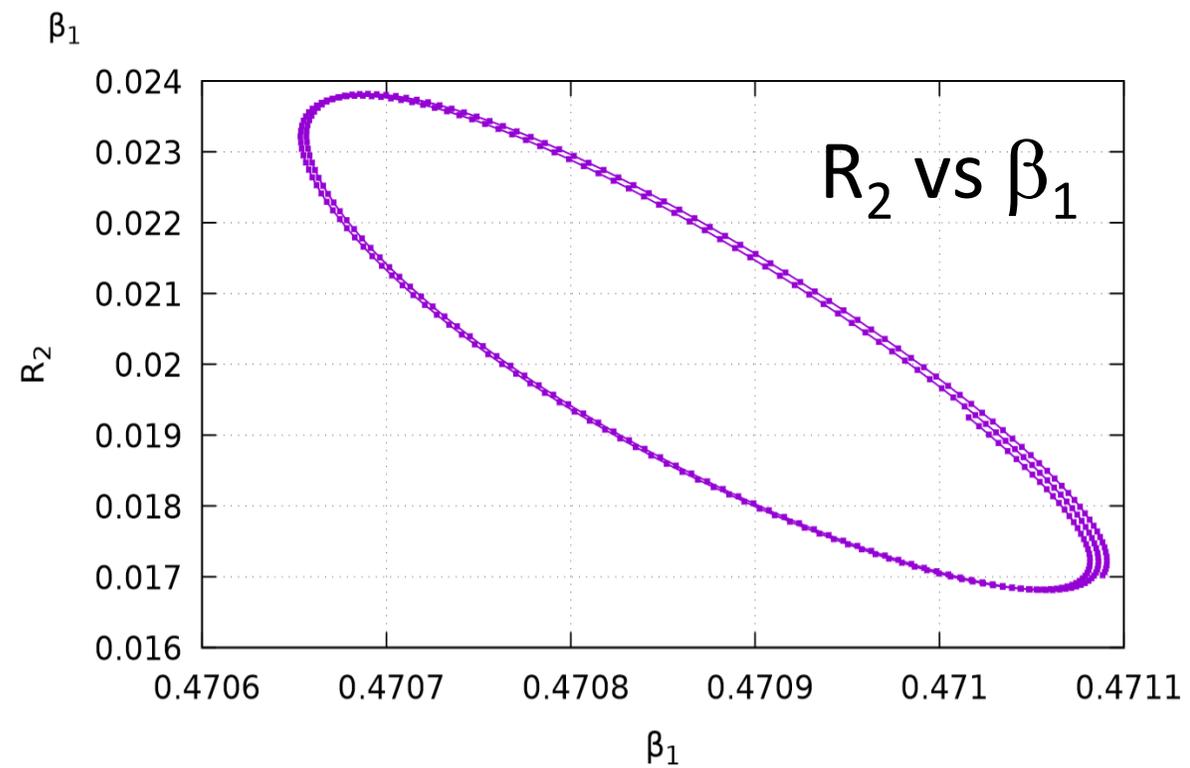
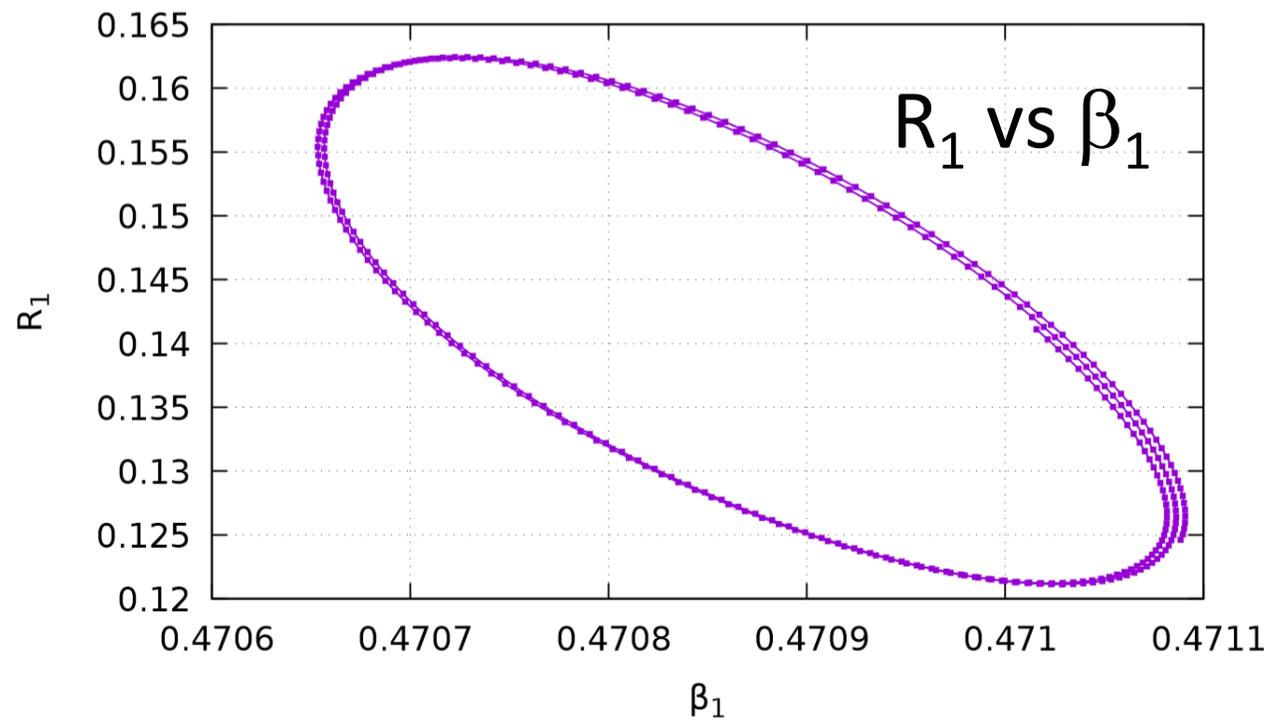
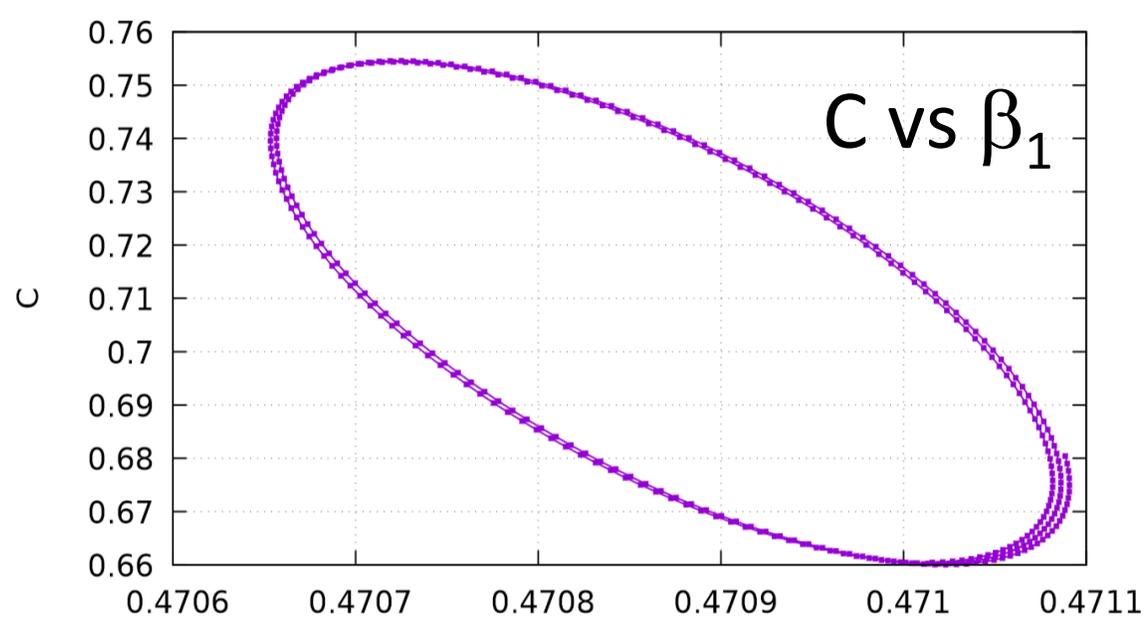
$m = 0.20, \alpha = (0.95, 0.065, 0.065, 0.95), \varepsilon = 10^{-6}, \beta_{11} = 0.5, \text{wsp} = 0.001300$

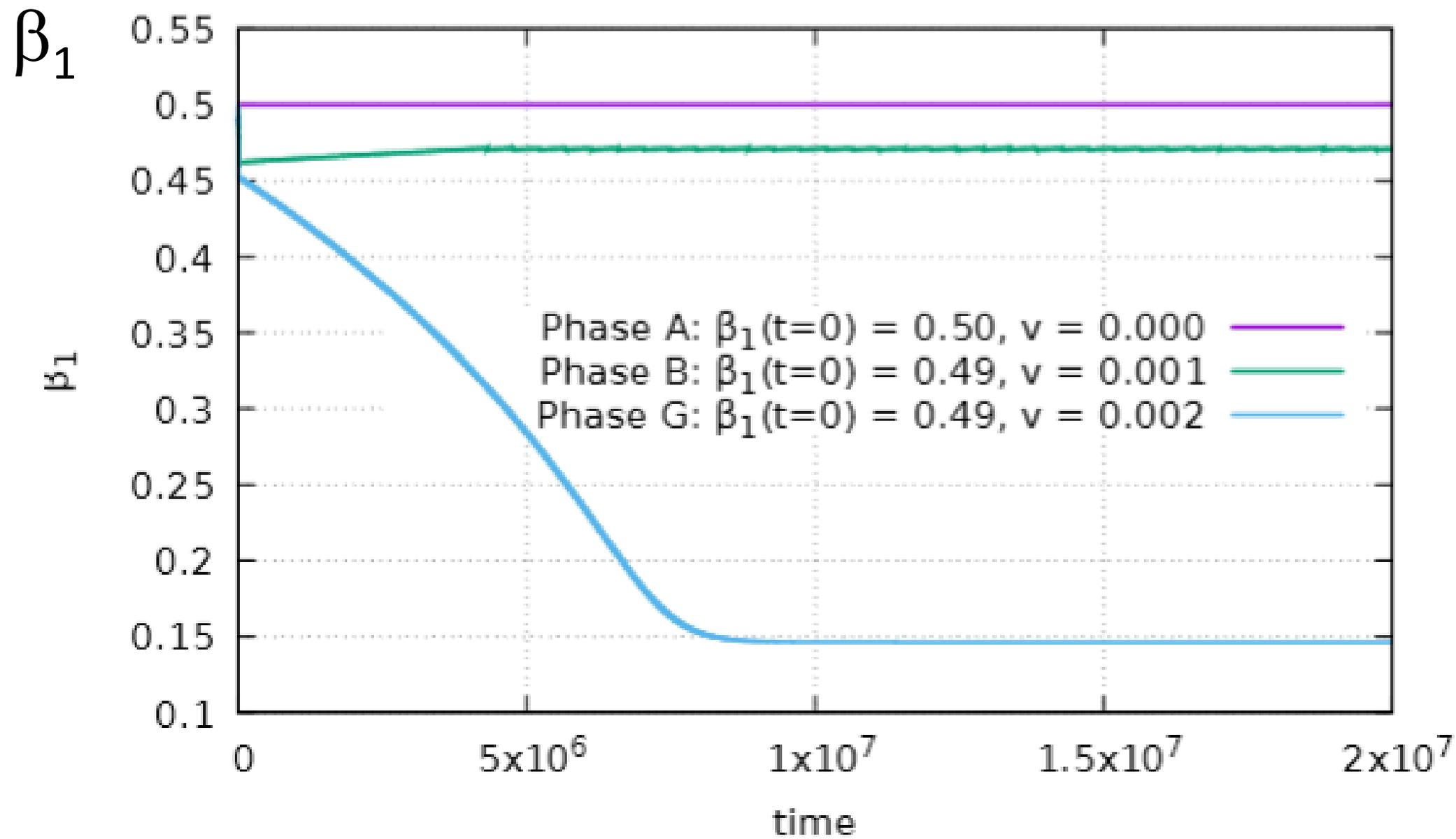




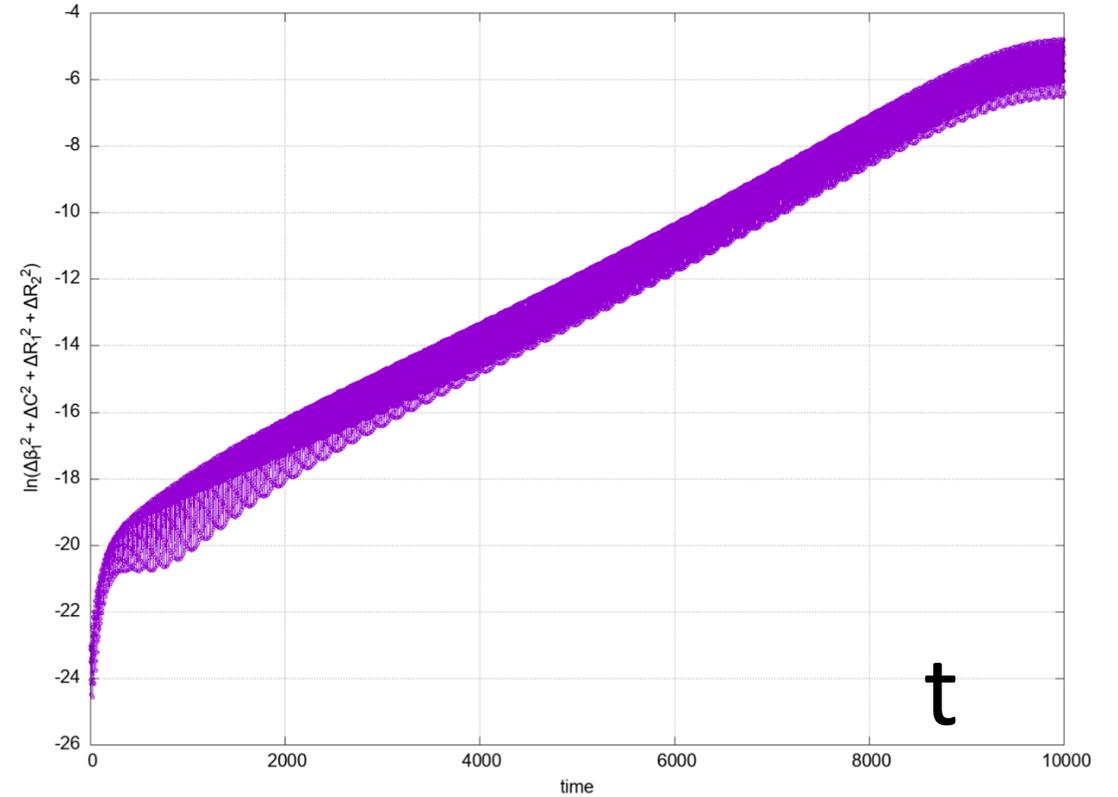
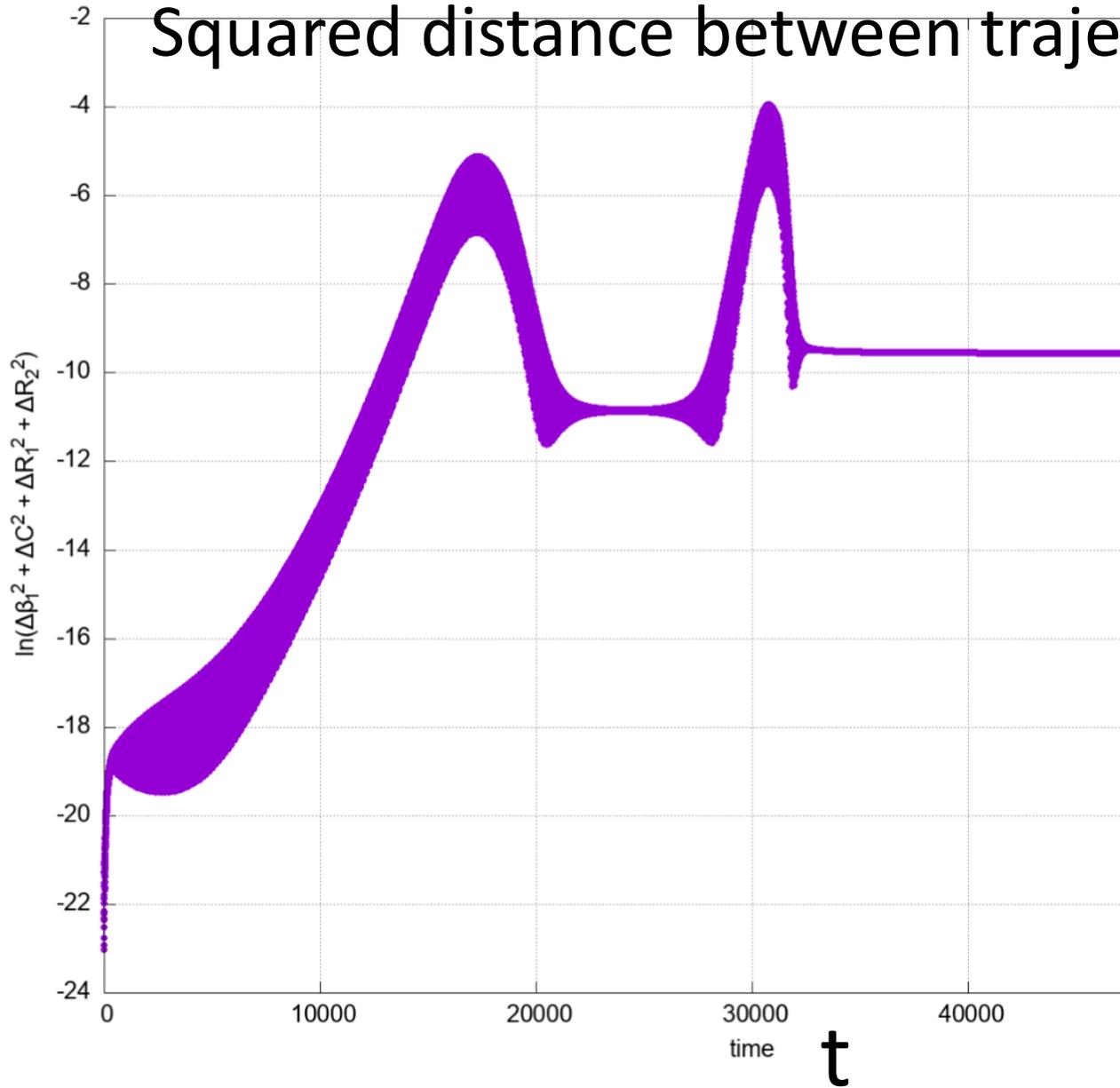
β_1 vs time



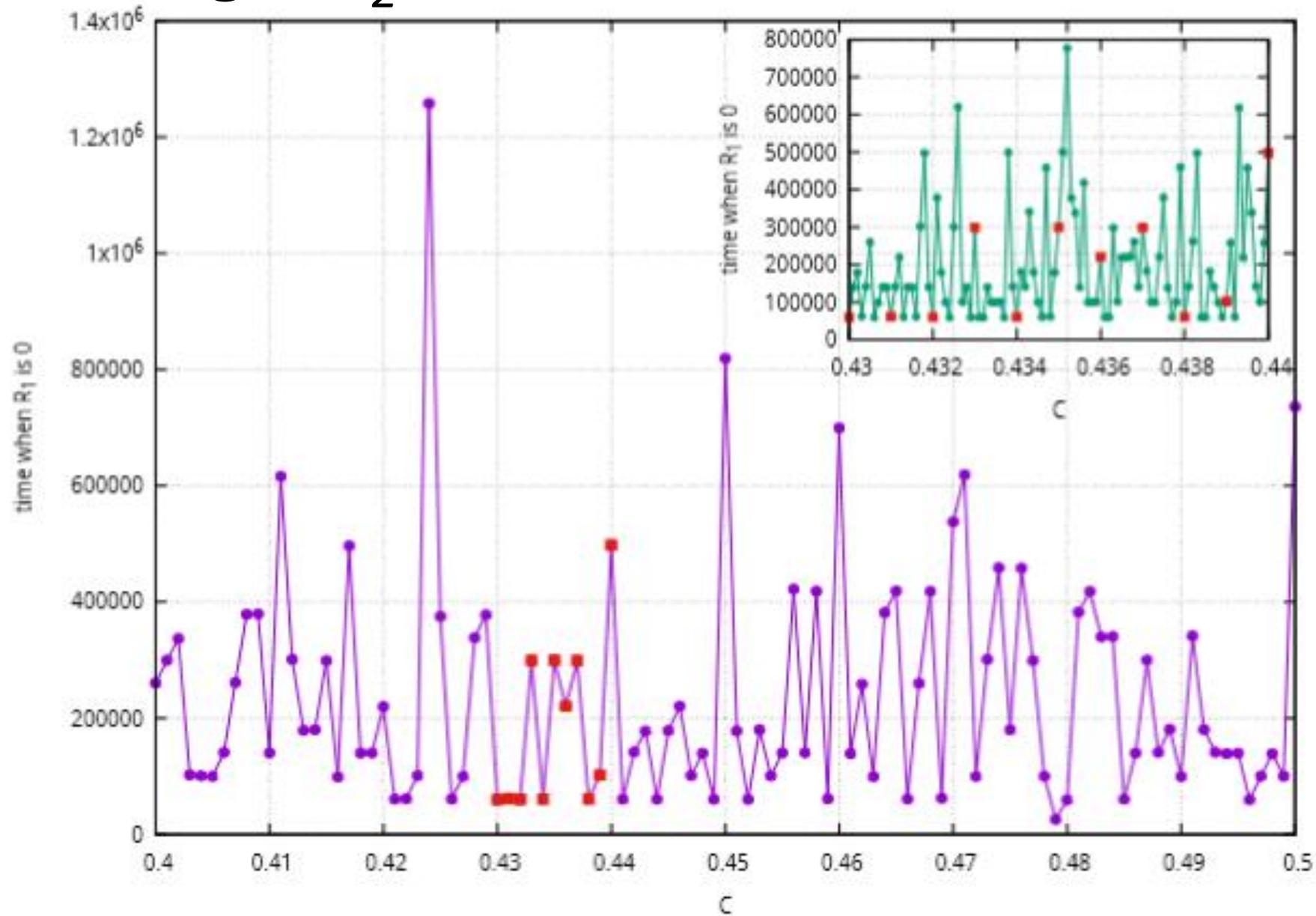




Squared distance between trajectories in 4-dimensional space



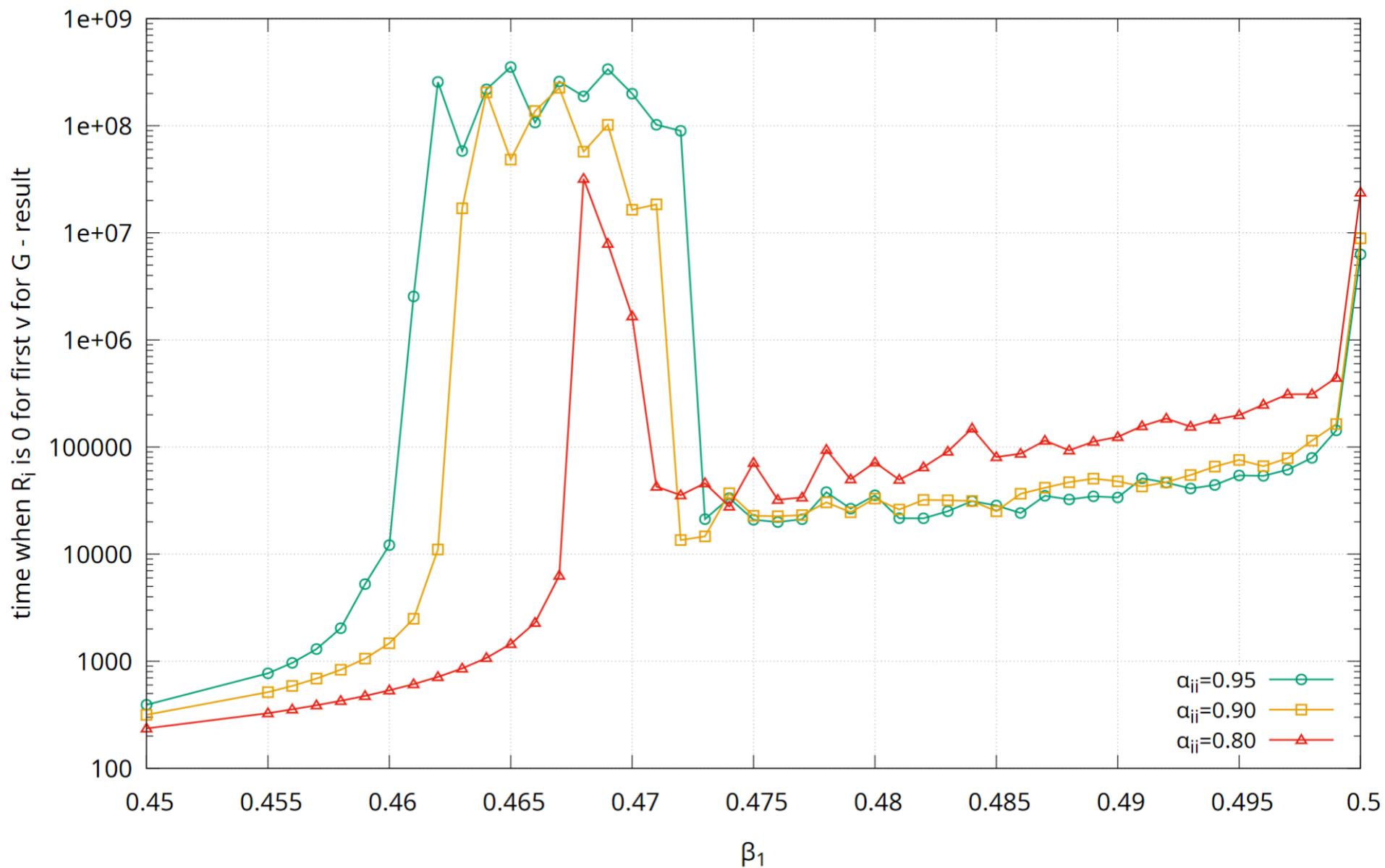
time to get $R_2=0$



$C(t=0)$

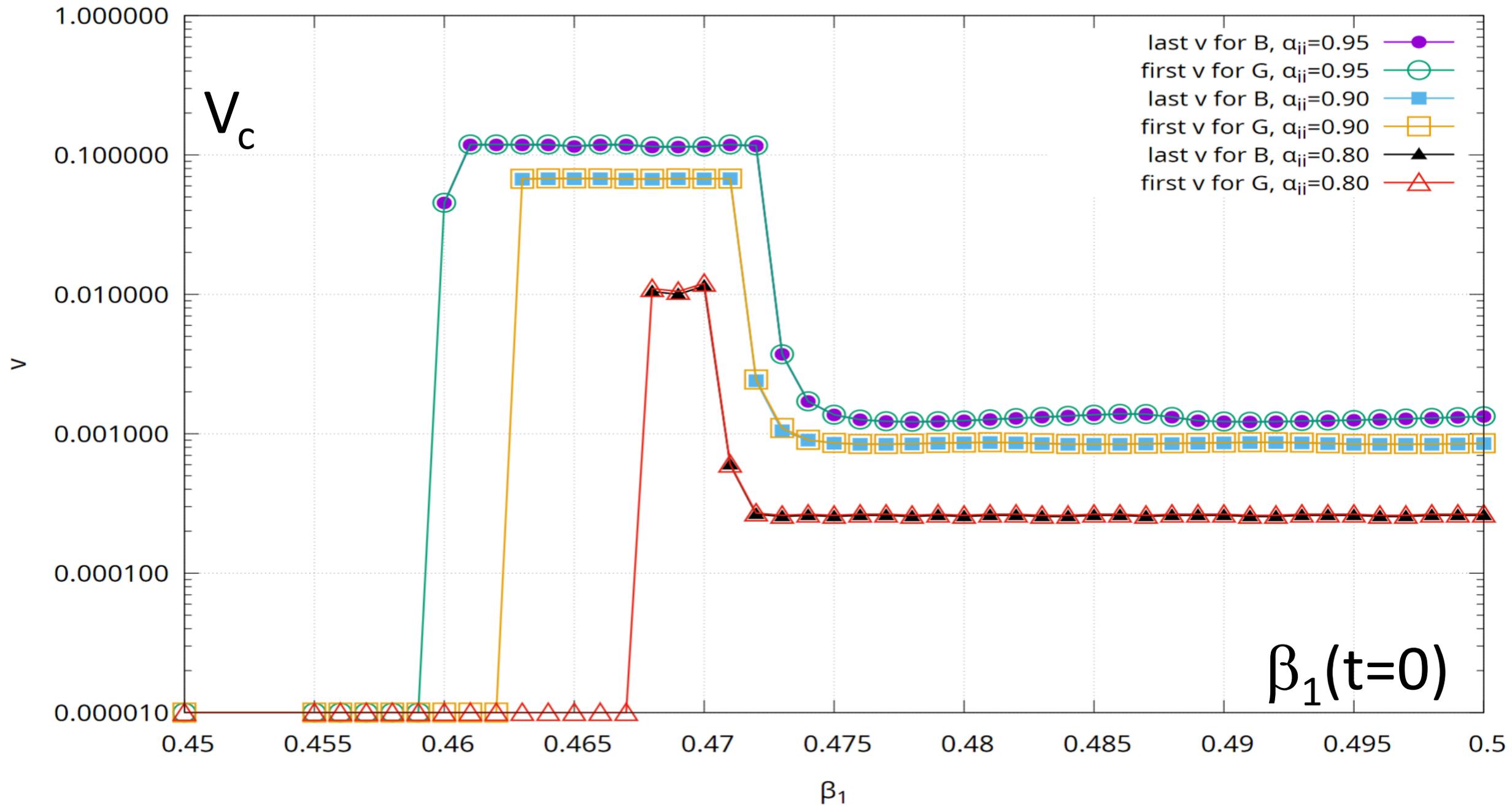
time to get $R_2=0$

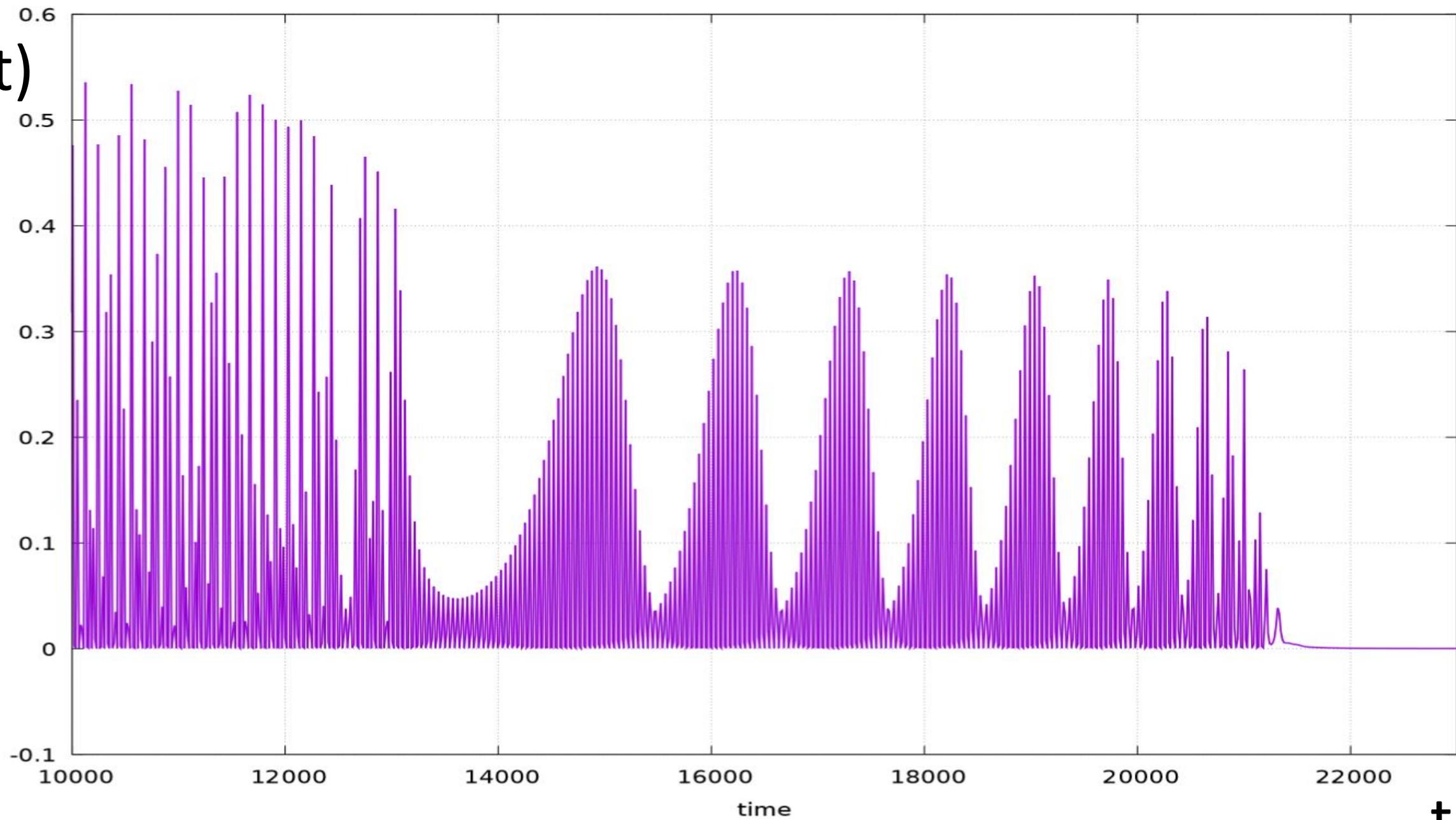
$\alpha=(\alpha_{ij}, 0.065, 0.065, \alpha_{ij})$



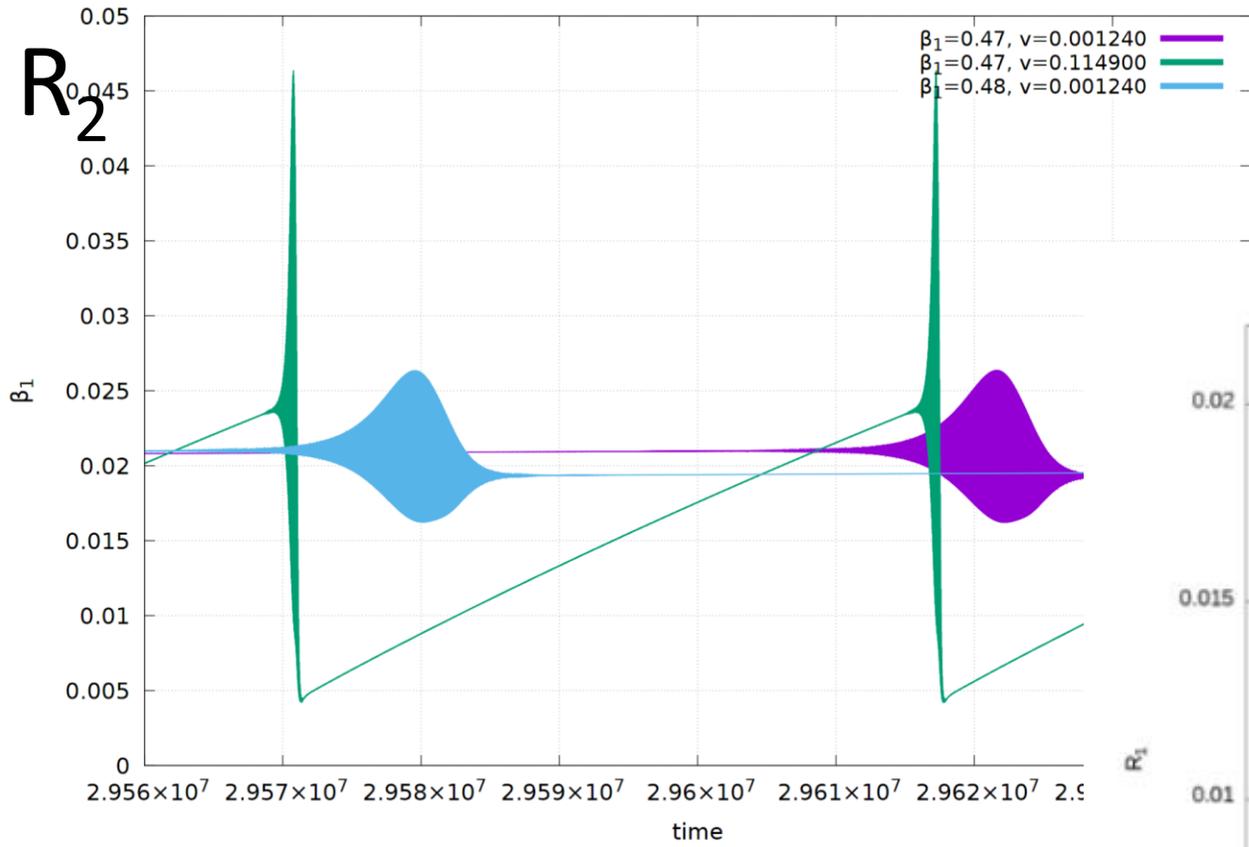
$\beta_1(t=0)$

$$\alpha = (\alpha_{ij}, 0.065, 0.065, \alpha_{ij})$$

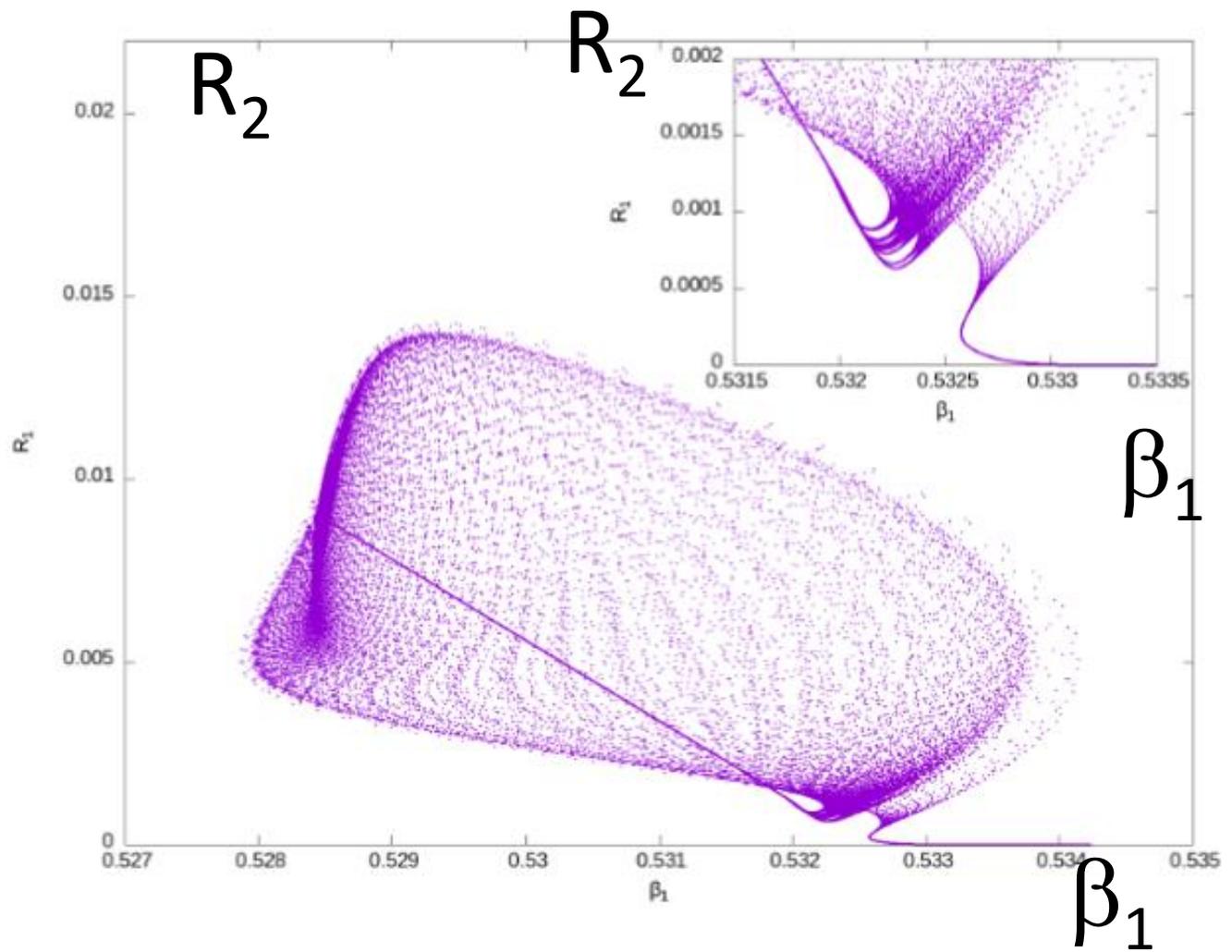


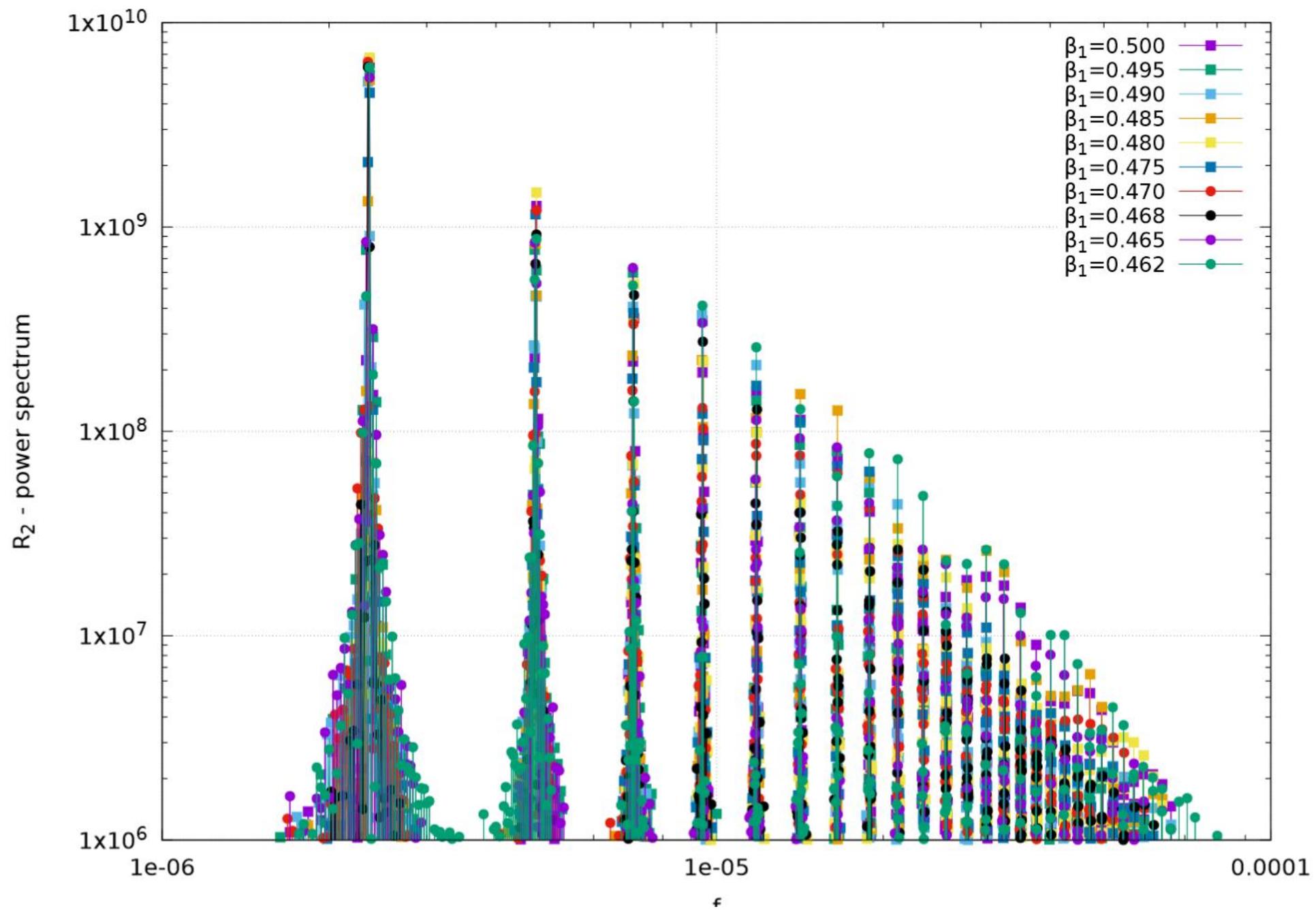
$R_2(t)$ R_2  t

$\alpha_{ij}=0.95$

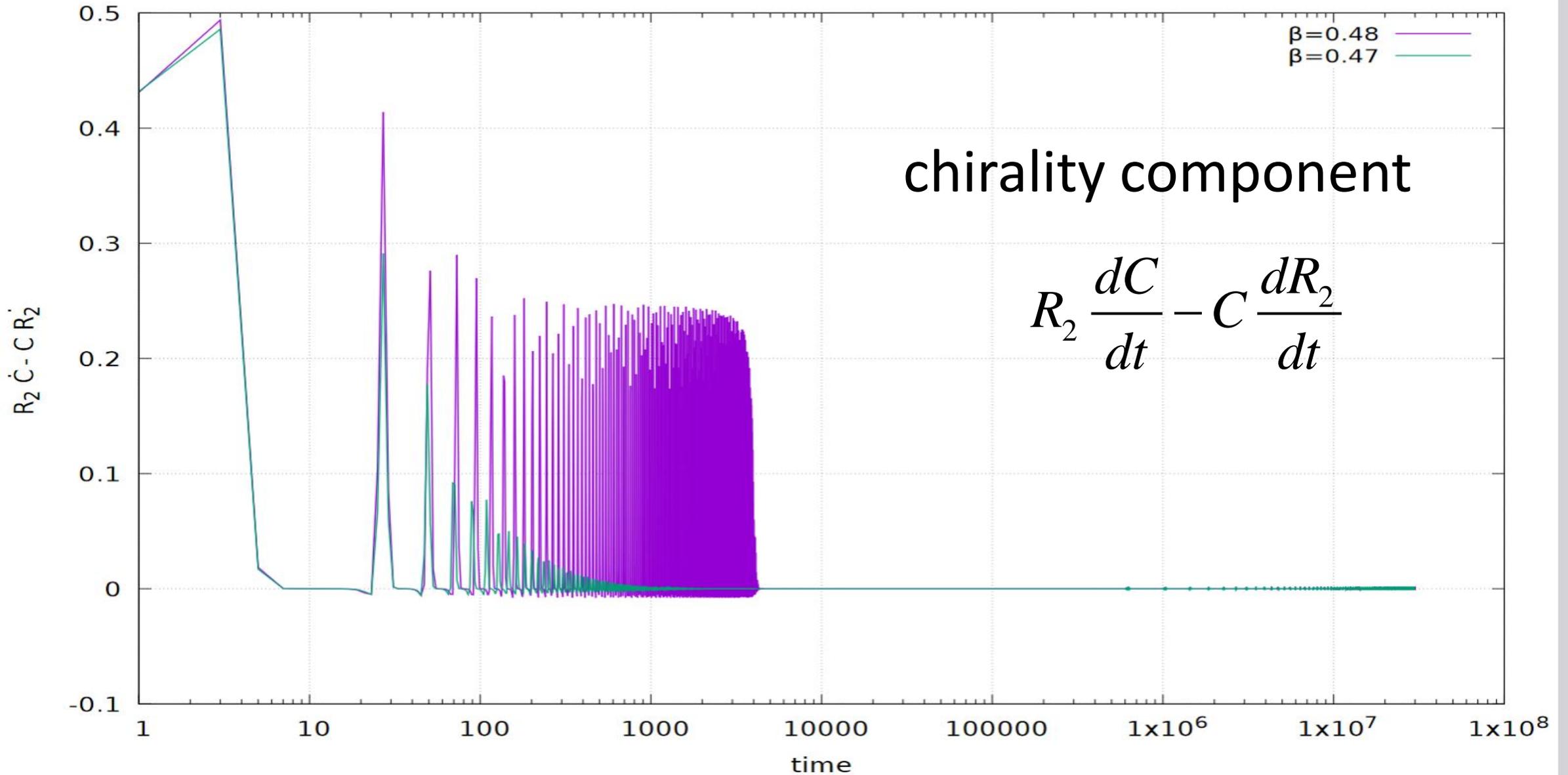


t

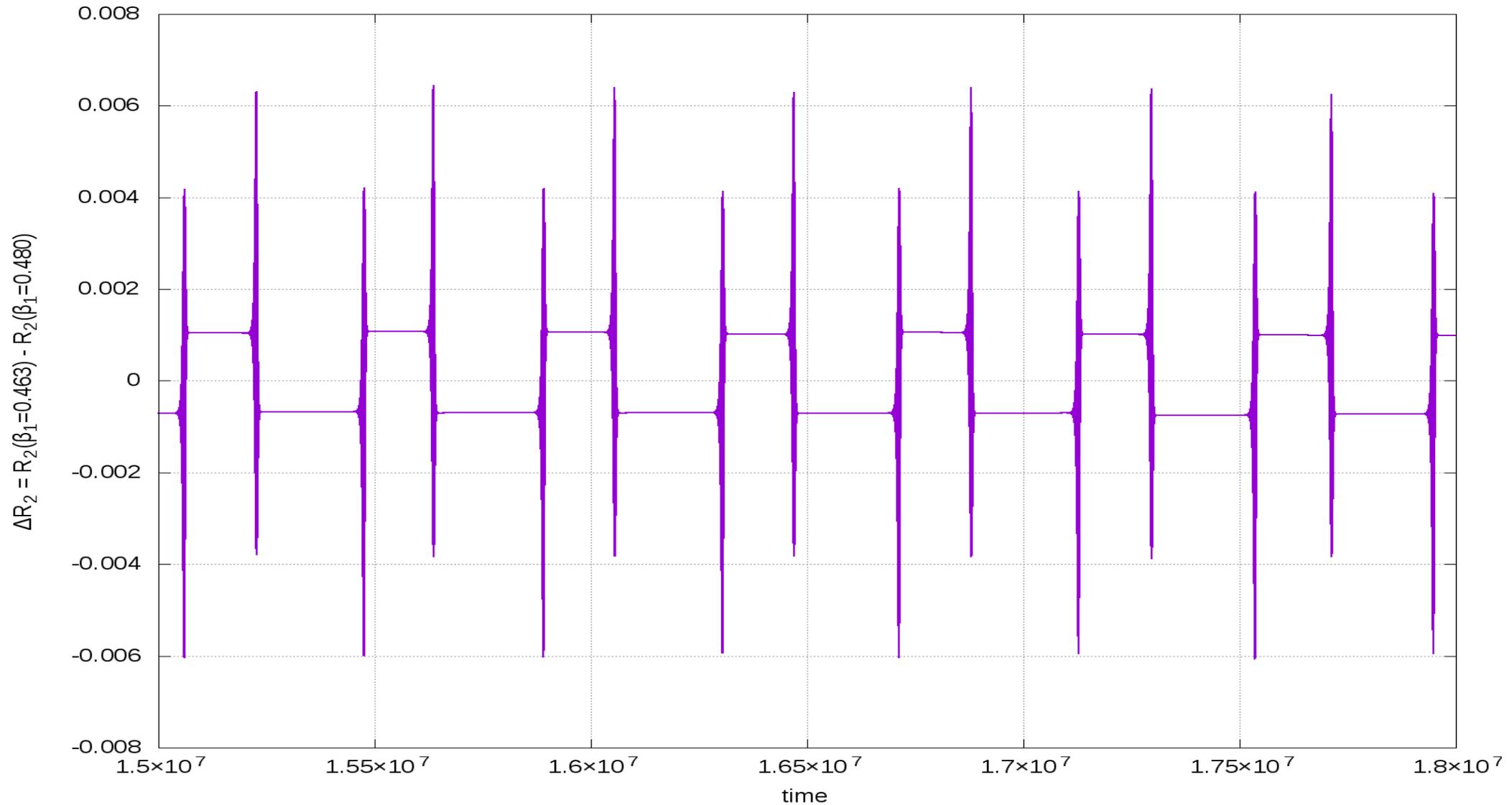




$\alpha=(0.95, 0.065, 0.065, 0.95), \nu=0.001240$

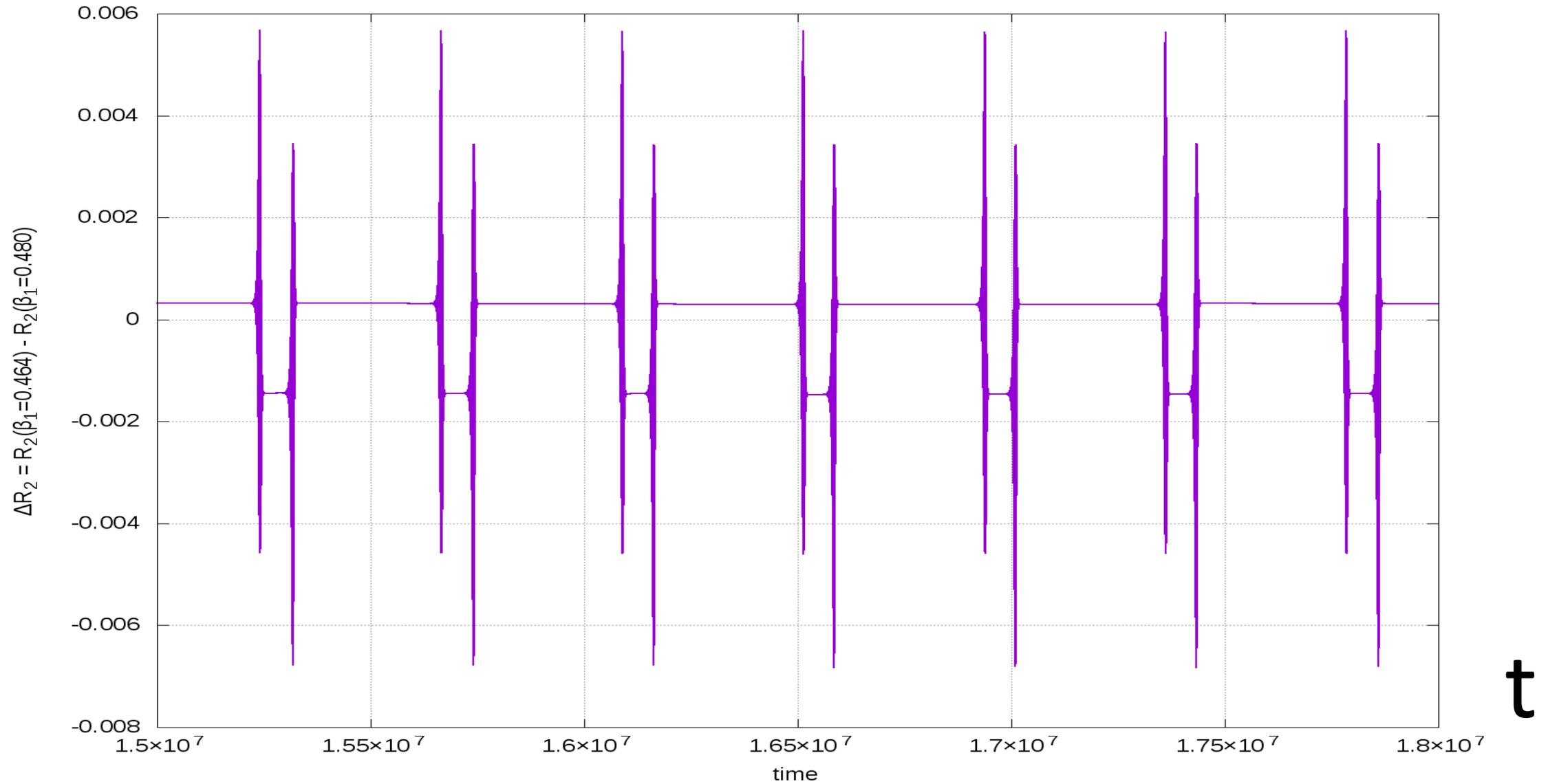


$$R_2(\beta_1(t=0) = 0.463) - R_2(\beta_1(t=0) = 0.480)$$



t

$$R_2(\beta_1(t=0) = 0.464) - R_2(\beta_1(t=0) = 0.480)$$



There are three phases:

- A (symmetric, $v=0$, $R_1 = R_2 > 0$),
- B (asymmetric, small v , $R_1 > R_2 > 0$ or $R_2 > R_1 > 0$)
- G (absorbing, large v and ($R_1 > 0$, $R_2 = 0$ or $R_1 = 0$, $R_2 > 0$)).

The time evolution of all variables (C , R_i , β_1) in phase B contains a initial transient chaotic stage and – later - periodic orbits. In the phase G, these variables do not depend on time.

The transition point v_c between B and G **strongly** depends on the initial value of the consumption rate β_1 . There are three modes of the transition:

- directly from the transient disorder of B to G (small v_c)
- from the periodic stage of B to G (large v_c)
- immediate.

The initial value of the rate β_1 influences the phase shifts of the periodic orbits.

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- [3] P. Gawroński, A. Borzi and K. K., Instability of oscillations in the Rosenzweig-MacArthur model of one consumer and two resources, *Chaos* 32 (2022) 093121.
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- [5] M. Begon, J. L. Harper and C. R. Townsend. Ecology: Individuals, populations, and communities. 2. Blackwell Sci. Pub.; Boston MA: 1990.